Global Climate Forum

Coastal impact modelling with the diva++ library

Session 1 - 17/08/23

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Organisatorisches

- Seminar website: https://globalclimateforum.org/diva_modelling/
- Sessions: Thursday 11.30 am CET at GCF meeting room
- Online participation: https://us02web.zoom.us/j/7893611627
- Manuscript and slides can be downloaded
- Exercise sheet available after seminar session. Voluntary but useful. Exercises will have a theory part and a practical part.

Requirement / Expectation management

- Required: basic math
- Required: basic programming
- Not provided: No teaching of programming - there are a lot of tutorials around

Literature:

- Herbert B. Enderton. Elements of Set Theory. Academic Press, 1977.

Basic Definitions

Set of numbers (no discussions of technicalities of philosophy of set theory here):

- $\mathbb{N}, \mathbb{Z}, \mathbb{R}, \mathbb{B}$ stand for the natural, integer and real numbers and for boolean values.
- $\mathbb{Z}_+, \mathbb{R}_+$ stand for the postive integer and real numbers (does not include zero).
- $\mathbb{Z}_{+,0}, \mathbb{R}_{+,0}, \mathbb{N}_0$ stand for the postive integer and real numbers including zero and the natural numbers including zero.
- $\perp$ denotes a special value, called "no data", "null", "void", "undefined", "bottom", etc.
- For any set S: $S_\perp = S \cup \{\perp\}$. Example: $\mathbb{R}_{+,0,\perp}$ denotes the set of real numbers greater or equal zero including $\perp$.

Basic Definitions - grids

- An infinite discrete (rectangular) grid is the set $G^{\infty} = \mathbb{Z} \times \mathbb{Z}$.
- Note that $\mathbb{N} \times \mathbb{N} \subset \mathbb{Z} \times \mathbb{Z}$ is an infinite grid (might be called one-side bounded).
- A finite discrete (rectangular) grid $G^{n \times m}$ is a finite subset of G^{∞}, i.e. $G^{(n,m)} = \{0, \ldots, n-1\} \times \{0, \ldots, m-1\}$. Here n and m stand for the number of columns and rows and it is said the G has the dimension $n \times m$.
- The elements $(i,j) \in G$ (G without any superscript stands for both infinite and finite discrete grids) are called grid cells (synonym is grid points).
- In (i,j) $\quad i$ denoted the row and j the column.
- We adopt here the technical notion inspired by the image processing: the upper left corner of any finite grid with dimension $n \times m$ is is the grid cell (0,0), the lower left corner is the grid cell $(0, m-1)$ and the upper right corner is the grid cell $(n-1, 0)$.

Basic Definitions - grids

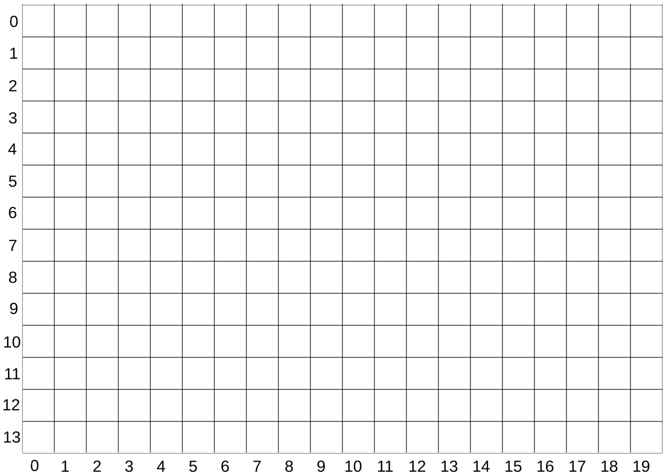

Figure: A finite grid with dimension 20×14.

Basic Definitions - grids, indices function

- We further denote for a finite discrete grid $G^{n \times m} = \{0, \ldots, n-1\} \times \{0, \ldots, m-1\}$ the indices function as follows:

$$cind_G(i,j) = \begin{cases} \emptyset & \text{if } i < 0 \text{ or } i \geq n-1 \\ \emptyset & \text{if } j < 0 \text{ or } j \geq m-1 \\ \{(i,j)\} & \text{otherwise} \end{cases}$$

For an infinite discrete grid G^{∞} the indices function is defined similarly (the $\emptyset$ cases do not exist then).

Basic Definitions - Neighbourhoud

- For a discrete grid G the 4-neighbourhoud of a grid cell (i, j) is defined as

$$\begin{aligned} M_G^4(i,j) &= cind_G(i-1,j) \cup cind_G(i+1,j) \\ &\cup cind_G(i,j-1) \cup cind_G(i,j+1) \end{aligned}$$

- Note: the pair (i, j) itself is not part of its own neighbourhoud. Additional note: $\forall G, i, j : 0 \leq \left| M_G^4 \right| \leq 4$
- The 4-neighbourhoud is also referred to as Von Neumann neighborhoud.

Basic Definitions - Neighbourhoud

- In the same way the 8-neighbourhood of a grid cell $(i,j) \in G$ is defined as

$$\begin{aligned} M_G^8(i,j) = & \quad cind_G(i-1,j) \cup cind_G(i+1,j) \\ & \cup\, cind_G(i,j-1) \cup cind_G(i,j+1) \\ & \cup\, cind_G(i-1,j-1) \cup cind_G(i+1,j-1) \\ & \cup\, cind_G(i-1,j+1) \cup cind_G(i+1,j+1) \end{aligned}$$

- Again, the pair (i,j) itself is not part of its own neighbourhood. It should be noted that $\forall G, i, j : 0 \leq \left| M_G^8 \right| \leq 8$
- Afterwards we always refer to the 8-Neighbourhoud unless stated otherwise.

Basic Definitions - Neighbourhoud

- For any grid G the neighbourhoud with radius r ($r > 0$) of a grid cell (i,j) is defined as

$$N_G^r(i,j) = \left(\bigcup_{k=-r}^{r} \bigcup_{l=-r}^{r} cind_X(i+k, j+l) \right) \setminus (i,j)$$

- From this definition and the definition above it follows that $N_G^1(i,j) = M_G^8(i,j)$.
- Any of the Neighbourhouds NH defined above we can be seen as a relation: $(i_1, j_1) NH (i_2, j_2) \iff (i_1, j_1) \in NH(i_2, j_2)$ For the properties of these relations see exercises.

Basic Definitions - Neighbourhoud

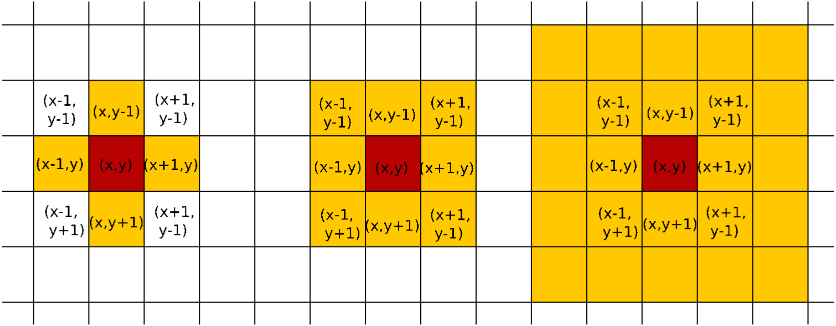

Figure: Illustration of the 4-neighbourhoud (left) and the 8-neighbourhoud (middle) and the Neighbourhoud with radius 2 of a grid cell.

Basic Definitions - Gridded datasets

- A gridded dataset on a (finite or infinite) discrete grid G is a mapping
$$d : G \rightarrow M_\perp$$
where M is the set of possile data values.
- In the same way a gridded multivariate dataset on a (finite or infinite) discrete grid G is a mapping
$$d : G \rightarrow M_{1,\perp} \times \ldots \times M_{k,\perp}$$
where $k \geq 1$ is the dimension of the the dataset.
- More accurate: A gridded dataset is the tuple $(G, M_\perp, d)$.
- for each gridded dataset the function *grid* strips the grid from the dataset

Basic Definitions - Gridded datasets

- For a gridded dataset *data* on a grid $G^{n \times m}$ the 4-neighbourhoud values are defined as

$$data_G^4(i,j) = \{data(i,j) \mid (i,j) \in M_G^4(i,j)\}$$

- in a similar way the 8-neighbourhoud values ($data_G^8(i,j)$) and the neighbourhoud values with radius r ($N_{data,G}^r(i,j)$) are defined.

Basic Definitions - Gridded datasets, examples

- A discrete digital elevation model (DEM) on a (finite) discrete grid G is a gridded dataset

$$DEM : G \to \mathbb{R}_\perp$$

The values $DEM(i,j)$ are interpreted as the elevation of the grid cell at position (i,j) relative to some elevation reference. If $DEM(i,j) = \perp$, the grid cell does not represent land (but rather ocean). The nature of the DEM can be further specified, e.g. as a digital surface elevation model or a digital ground elevation model.

- A discrete digital coastal surface elevation and distance model $DSEDM$ on a (finite) discrete grid G is a gridded dataset

$$DSEDM : G \to \mathbb{R}_\perp \times \mathbb{R}_{+,0,\perp}$$

where the values $DSEDM(i,j)$ are pairs (e,d) where e is interpreted as the elevation of grid cell (i,j) and d is interpreted as the distance of grid cell (i,j) to the coastline.

Basic Definitions - Connectivity

- For a gridded dataset *data* on a grid $G^{n \times m}$ two grid cells (i_s, j_s) and (i_e, j_e) are defined to be 4-connected if there exists a sequence of grid cells $p = (gc_1, \ldots, gc_n) = ((i_1, j_1), \ldots, (i_n, j_n))$ such that
  - $gc_1 = (i_s, j_s)$
  - $gc_n = (i_e, j_e)$
  - $k1 \neq k2 \ \rightarrow gc_{k1} \neq gc_{k2}$
  - $\forall \ gc_k = (i_k, j_k) : data(i_k, j_k) \neq \perp$
  - $\forall \ gc_k = (i_k, j_k)$ with $k > 1 : (i_k, j_k) \in M_G^4(i_{k-1}, j_{k-1})$
- The sequence p is called a 4-connected path from $g_1 = (i_s, j_s)$ to $g_n = (i_e, j_e)$. In a similar way two grid cells (i_s, j_s) and (i_e, j_e) are defined to be 8-connected.

Basic Definitions - Connectivity

- Translation of the requirenments of a X-connected path:
  - $gc_1 = (i_s, j_s)$ - start grid cell should be first element of the path
  - $gc_n = (i_e, j_e)$ - end grid cell should be first element of the path
  - $k1 \neq k2 \ \rightarrow gc_{k1} \neq gc_{k2}$ - a path is not supposed to visit the same grid cell twice. This means a path is loop-free.
  - $\forall\ gc_k = (i_k, j_k) : data(i_j, j_k) \neq \perp$ - a path only visits data grid cells
  - $\forall\ gc_k = (i_k, j_k)$ with $i > 1 : (i_k, j_k) \in M_G^X(i_{k-1}, j_{k-1})$ - subsequent path grid cells are neighbours

Basic Definitions - Constrained Connectivity

- For a gridded dataset *data* on a grid $G^{n \times m}$ and a predicate $pred : (M_\perp)^k \rightarrow \mathbb{B}$ two grid cells (x_s, y_s) and (x_e, y_e) are defined to be *pred*-constrained 4-connected if there exists a sequence of grid cells $p = (gc_1, \ldots, gc_n)$ such that
  - p is a four-connected path from gc_1 to gc_n.
  - $\forall\ gc_k = (i_k, j_k) : pred(data(i_k, j_k)) = true$

  The sequence p is called a *pred*-constrained 4-connected path from (i_s, j_s) to (i_e, j_e). In a similar way two grid cells (i_s, j_s) and (i_e, j_e) are defined to be *pred*-constrained 8-connected.
- For any path $p = (gc_1, \ldots, gc_n)$ the value $max_{data}(p) = max\{data(gc_1), \ldots, data(gc_n)\}$ is called the maximal data value of p.
- Any connectivity *Con* defines a relation: $(i_1, j_1) Con (i_2, j_2) \iff (i_1, j_1)$ and (i_2, j_2) are Con-connected. See exercises for properties of these relations.
- Example for a often used predicate: $\leq_\theta (v) = v \leq \theta$

Basic Coastal Definitions - Coastal Model

Definition

In a discrete DEM a grid cell $g = (i, j)$ is called coastline iff $DEM(i, j) \neq \perp \ \wedge \ \perp \in DEM^8_G(i, j)$. That is, the coastline is the set of grid cells that contain a data value and at least one no data value in their 8-neighbourhoud.

Basic Coastal Definitions - Coastal Model

0	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	2.0	2.2	0.9	1.1	1.4	1.4	1.3	⊥	⊥	⊥
2	⊥	2.5	3.3	2.2	1.4	1.0	0.9	1.2	0.7	0.8	⊥
3	⊥	2.8	3.0	4.2	5.6	0.7	0.8	2.0	1.6	0.6	⊥
4	⊥	⊥	3.2	3.7	4.2	0.6	0.5	1.2	1.5	0.8	⊥
5	⊥	⊥	2.9	2.2	1.5	1.2	1.4	1.2	⊥	⊥	⊥
6	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥
	0	1	2	3	4	5	6	7	8	9	10

Figure: Illustration of an DEM

Basic Coastal Definitions - Coastal Model

	0	1	2	3	4	5	6	7	8	9	10
0	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	2.0	2.2	0.9	1.1	1.4	1.4	1.3	⊥	⊥	⊥
2	⊥	2.5	3.3	2.2	1.4	1.0	0.9	1.2	0.7	0.8	⊥
3	⊥	2.8	3.0	4.2	5.6	0.7	0.8	2.0	1.6	0.6	⊥
4	⊥	⊥	3.2	3.7	4.2	0.6	0.5	1.2	1.5	0.8	⊥
5	⊥	⊥	2.9	2.2	1.5	1.2	1.4	1.2	⊥	⊥	⊥
6	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥

Figure: Illustration of an DEM with water (blue)

Basic Coastal Definitions - Coastal Model

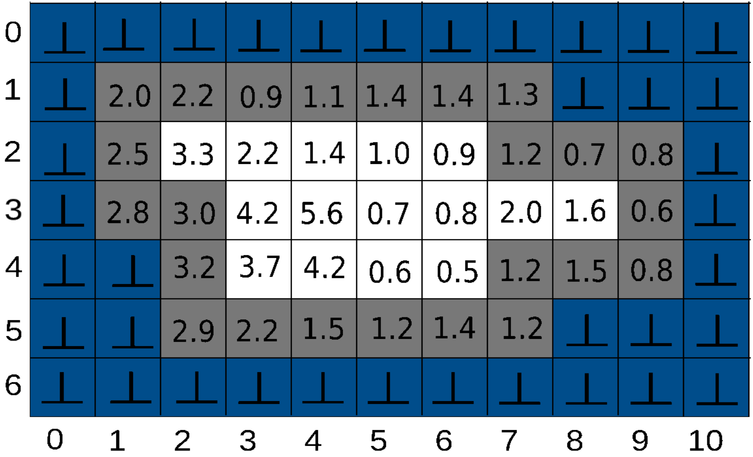

Figure: Illustration of an DEM with water (blue) and coastline (grey).

Basic Coastal Definitions - Coastal Model

```
extractCoastline (dem: DEM): Dataset
  var dat: Dataset
  for ((i,j) in grid(dem))
    if (dem(i,j) != nodata(dem))
      nh8 = Neighbourhoud8(dem,i,j)
      for ((i2,j2) in nh8)
        if(dem(i2,j2) == nodata(dem))
          insert(dat,i,j,dem(i,j))
          break
        end
      end
    end
  end
  return dat
end
```

Basic Coastal Definitions - Coastal Model

Definition

A Coastal Model (CM) cm is a two-variate dataset (as defined before)

$$cm : G \to \mathbb{B} \times \mathbb{R}_{\perp}$$

where G is a finite grid. In a coastal model the value $cm(i,j) = (c,e)$ is interpreted as follows: b is true if and only if the grid cell is coastline and e decodes the elevation. For a CM cm the following two projections are defined:

$$\begin{aligned} coastline : G &\to \mathbb{B} \\ coastline(i,j) &= c \iff cm(i,j) = (c,e) \\ elevation : G &\to \mathbb{R}^{\perp} \\ elevation(i,j) &= e \iff cm(i,j) = (c,e) \end{aligned}$$

Basic Coastal Definitions - Coastal Model

This definition can be generalized into an extended CM with arbitrary many datasets involved:

Definition

An (extended) CM cm is a multi-variate dataset (as defined before)

$$cm : G \to \mathbb{B} \times \mathbb{R}_\perp \times (\mathbb{R}_\perp)^n$$

where G is a finite grid and $n \in \mathbb{N}$ with $n \geq 0$ and further $cm(i,j) = (c, \perp, \vec{d}) \Rightarrow \vec{d} = \vec{\perp}$. In a coastal model the values c and e in $cm(i,j) = (c, e, \vec{d})$ are interpreted as before. The projection functions remain and I assume that there is a projection function (with a meaningful name) for each dimension of $\vec{d}$.

Basic Coastal Definitions - Coastal Model

- An (extended) CM can be seen as a combination of several gridded datasets (all operating on the same grid). The elevation dataset is given explicitly with the requirement that if elevation is no data then all other dimensions should also be no data.
- I do not distinguish between CM and extended CM anymore.
- The involved dimensions and projections should be clear from the context.

Basic Coastal Definitions - Coastal Model Example

Example

A simple coastal model for flood assessment could be defined as an extended CM $cm : G \to \mathbb{B} \times \mathbb{R}_\perp \times (\mathbb{R}_\perp)^4$ with the following projections:

$$
\begin{aligned}
coastline : G &\to \mathbb{B} \\
coastline(i,j) &= c \iff cm(i,j) = (c, e, d_1, d_2, d_3, d_4) \\
elevation : G &\to \mathbb{R}_\perp \\
elevation(i,j) &= e \iff cm(i,j) = (c, e, d_1, d_2, d_3, d_4) \\
area : G &\to \mathbb{R}_{+,0,\perp} \\
area(i,j) &= d_1 \iff cm(i,j) = (c, e, d_1, d_2, d_3, d_4) \\
population : G &\to \mathbb{R}_{+,0,\perp} \\
population(i,j) &= d_2 \iff cm(i,j) = (c, e, d_1, d_2, d_3, d_4) \\
assets : G &\to \mathbb{R}_{+,0,\perp} \\
assets(i,j) &= d_3 \iff cm(i,j) = (c, e, d_1, d_2, d_3, d_4) \\
distance : G &\to \mathbb{R}_\perp \\
distance(i,j) &= d_4 \iff cm(i,j) = (c, e, d_1, d_2, d_3, d_4)
\end{aligned}
$$

Basic Coastal Definitions - Coastal Zones

Starting from a CM *cm* coastal zones and flood plains can be defined.

Definition

For a *cm* the coastal zone with elevation threshold θ_{el} ($cz(cm, \theta_{el})$) is defined as

$$
\begin{aligned}
cz(cm,\theta_{el}) \quad &: \quad G \rightarrow \mathbb{R}_{\perp} \\
cz(cm,\theta_{el})(i,j) \quad &= \quad \begin{cases} el & \text{if } elevation(i,j) = el \text{ and} \\ & \exists (i_2, j_2) \text{ s.t. } coastline(x_2,y_2) = true \text{ is} \\ & \leq_{\theta_{el}} \text{-constrained 8-connected to } (i,j) \\ & \text{where } \leq_{\theta_{el}} (e) = e \leq \theta_{el} \\ \perp & \text{otherwise} \end{cases}
\end{aligned}
$$

Basic Coastal Definitions - Coastal Zones

- That is, $cz(cm, \theta_{el})$ is a dataset consisting from the same grid as the underlying CM *cm* mapping the original elevation value to all grid cells that are 8-connected to the coastline with a path for that all grid cells on the path have elevation of at most θ_{el}.
- special coastal zones with elevation threshold: the low elevation coastal zone (LECZ) is the coastal zone with elevation threshold 10.0m, the extended low elevation coastal zone (ELECZ) is the coastal zone with elevation threshold 20.0m.
- The coastal zones itself can be made part of the CM by introducing dimensions and projections for them.
- Or by replacing the elevation values by the coastal zone elevation values (setting none-coastal-zone values to $\perp$).

The end

Thanks.