

Vol. 1 • No. 11 • November • 2018

www.advtheorysimul.com

ADVANCED THEORY AND SIMULATIONS

WILEY-VCH

Systemic Risks: Theory and Mathematical Modeling

Klaus Lucas, Ortwin Renn,* and Carlo Jaeger*

In a globally connected world, new opportunities are associated with new types of risks. These new types of risks do not respect national boundaries nor are they restricted to particular locations or systems. Instead, they are characterized by contagion and proliferation processes, frequently on the basis of a network structure, with the result that a seemingly harmless local event is able to cause a complete system collapse. The proposal has been made to refer to these types of new risks as systemic risks. It turns out that key phenomena associated with systemic risks can quite naturally be categorized and analyzed in terms of notions originally established in the natural sciences, such as those of chaos, order, and self-organization, or, more concisely, of dynamic structure generation in complex open systems. In this Essay, the claim is made that there is a homomorphism within the dynamic structure generation across very different domains of systemic risks. Furthermore, there are structural similarities between complex structures in general, and systemic risks in particular. Based on this assumption, one can use established methodologies of complexity science to reveal general macroscopic patterns that seem to govern the dynamics of complex systems.

1. Introduction

The history of the last four decades has been a success story in terms of conventional risk management. Conventional risks are bounded in time and space, they can be well described in terms of functional relationships between extent of expected losses and probability of occurrence.^[1] Most importantly, conventional risks are not highly complex (however often complicated to understand), they are not highly interconnected with other types of risks and they can be effectively managed by technical and organizational measures. The success of conventional risk

management is documented in many statistical data. If one takes the example of Germany, the home country of the authors, the number of fatal accidents at work decreased from almost 5000 in 1960 to less than 500 in 2014; the number of traffic accidents from 22 000 in 1972 to 3700 in 2014, the number of fatal heart attacks and strokes decreased from 109 cases per 100 000 to 62 in the time period between 1992 and 2002.^[2] In addition, the number of chronic illnesses as well as fatal diseases from environmental pollution or accidents steadily declined over the past three decades.

The picture becomes, however, less favorable if one looks at globally interconnected, nonlinear risks such as those posed, for example, by climate change or the global financial system and the closely related growing inequality between rich and poor. In order to take account of this situation, the Organisation for Economic Co-operation and Development (OECD) introduced the new category of “systemic risk”.^[3] A widely

cited definition of a systemic risk was provided by Kaufman und Scott (2003). “Systemic risk refers to the risk or probability of breakdowns in an entire system, as opposed to breakdowns in individual parts or components, and is evidenced by co-movements (correlation) among most or all parts”.^[4] This definition assumes that the targeted system represents a vital service to society such as energy or financial stability. If we take a simple device such as a car or a sewing machine, a total collapse of such a technological device would not qualify as a systemic risk (Renn et al.^[5]). Hence, systemic risk refers to a potential collapse of a system of potentially global importance and criticality to services that humans urgently need. This dimension of a large potential threat within a complex web of interacting elements distinguishes systemic from other types of risk.

Renn et al.^[5] emphasize four major properties of systemic risks. They are: 1) transboundary, 2) highly interconnected and intertwined leading to complex causal structures, 3) nonlinear in the cause–effect relationships showing often unknown tipping points or tipping areas, and 4) stochastic in their effect structure. One of the main features of systemic risks include ripple effects beyond the domain in which the risks originally appeared and the threat of a multiple system breakdown that would affect multiple services to society. They may be felt in a wide range of systems seemingly well-buffered from each other, like the real estate and the health system, inflicting harm and damage in domains far beyond their own. Industrial sectors, for example, may suffer significant losses as a result of a systemic risk as we witnessed

Prof. K. Lucas
Institute of Technical Thermodynamics
Aachen University
Schinkelstraße 8, 52062 Aachen, Germany
Prof. K. Lucas, Prof. O. Renn, Prof. C. Jaeger
Institute for Advanced Sustainability Studies (IASS)
Berliner Str. 130, 14467 Potsdam, Germany
E-mail: ortwin.renn@iass-potsdam.de
Prof. C. Jaeger
Global Climate Forum
Neue Promenade 6, 10178 Berlin, Germany
E-mail: carlo.jaeger@globalclimateforum.org
Prof. C. Jaeger
Academy of Disaster Reduction and Emergency Management
Beijing Normal University
Beijing, China

DOI: 10.1002/adts.201800051

in the financial crisis in the aftermath of the Lehman Brothers collapse.

The main problem is that it is often difficult to predict when a system will suffer a breakdown or collapse. Threats to the system, such as climate change, may be hidden in small incremental effects that provide no hint about when thresholds have been reached. Or a collapse may occur due to a domino effect where a small glitch is released that affects multiple elements within a system or even multiple systems in parallel, thereby amplifying the overall risk.^[6]

Systemic risks pose serious challenges for risk assessment and risk management because they are not amenable to the reductionism of the standard risk assessment model. They require a more holistic approach to hazard identification, to risk assessment, and to risk management, because they are complex, stochastic, and nonlinear. This means that it is difficult to trace the connections between causes and effects, to understand the direct impacts of human actions against a background of random changes, and to start learning from simulation rather than from trial and error. Risk analysis for systemic risks must focus on interdependencies, ripple and spill-over effects, and other nonlinear dynamics that initiate impacts that cascade between otherwise unrelated risk domains. Governing systemic risks presents specific and unique challenges, challenges magnified by the reality that systemic risks vary considerably across and within systems; not two are exactly alike.

2. Theory

The notion of systemic risks is generic to describing phenomena of an entire breakdown of whole systems macroscopically due to reinforcing feedback actions of agents on the microlevel. Here, agents, in a most general sense, are conceptualized as elements of a system that interact among each other or with the environment. In technical systems, agents may be part of a technical infrastructure such as control and generation units in the electrical grid, the systemic risk would then be a breakdown of the grid as a whole. In ecosystems, agents such as harmful chemicals in interaction with a fish population in a river constitute the systemic risk of irreversible destruction of the population. In the global climate system, interacting agents comprise the sun radiation, clouds, carbon dioxide, and the surfaces of earth and water, which in conjunction with each other constitute the systemic risk of climate change. In social systems, humans are the agents interacting among themselves and with the system's environment, with systemic risks manifesting themselves in radical public movements up to social unrest and revolutions with the result, for example, of mass migration phenomena.^[7] The last example is instructive also because what may be a systemic risk for some agent may be an opportunity for others—consider, for example, the breakdown of a dictatorship or a Mafia racket. In the present contribution, we do not discuss the theory of how different agents may evaluate critical events but focus on the dynamics leading to such events.

It turns out that the rather diverse and chaotic elementary processes of the agents on the microlevel in the various domains surprisingly order themselves on the macrolevel to widely universal dynamic patterns, which can be formulated in terms of



Klaus Lucas is professor emeritus and former holder of the Chair for Technical Thermodynamics at Aachen University. His main fields of interest during his university career were molecular models for fluids, phase and reaction equilibria, and energy systems analysis. After the conclusion of his disciplinary research and teaching at the university, he served as vice president of the

Berlin–Brandenburg Academy of Sciences and Humanities, where he has turned to interdisciplinary research and project organization related to various topics including evolution theory, research program generation, and dynamic structures. He is presently a fellow of the Institute of Advanced Sustainability Studies in Potsdam.



Ortwin Renn is scientific director at the International Institute for Advanced Sustainability Studies (IASS) in Potsdam (Germany) and professor for environmental sociology and technology assessment at the University of Stuttgart. He also directs the non-profit company DIALOGIK, a research institute for the investigation of communication and participation processes. Renn is adjunct professor for “Integrated Risk Analysis” at Stavanger University (Norway), honorary professor at the Technical University Munich and affiliate professor for “Risk Governance” at Beijing Normal University. His research interests include risk governance (analysis perception, communication); stakeholder and public involvement in environmental decision-making; transformation processes in economics, politics, and society; and sustainable development.



Carlo Jaeger is a Swiss economist investigating global systemic risks like those of climate change and of today's financial markets in a perspective of green growth. He is cofounder and chairman of the Global Climate Forum, professor at Potsdam University, Germany, and works closely with Arizona State University and with Beijing

Normal University. He holds degrees in economics (Ph.D., Frankfurt University, Germany), sociology (diploma, University of Berne, Switzerland), and human ecology (habilitation, ETH Zurich, Switzerland). Jaeger has worked extensively on the interactions between technological progress and environmental problems, in particular, the role of digitalization in urban development.

simple macroscopic parameters. The dynamic evolution of these patterns, and thus the associated systemic risks, may be studied by differential equations, iterated functions, by agent-based modeling, and by complex network theory. These mathematical manifestations, while originally developed for model systems of physics and chemistry, seem to be universally valid in all domains, including the social sciences.

Clearly, when it comes to making quantitative predictions of a system's dynamic behavior, the differences between the various systems below the universal, conceptual, and mathematical superstructure become relevant. They originate from the specific interactions of the agents. While quantitative results can easily be found in some simple model systems of physics and chemistry, this is more of an exception in more general cases, notably the social sciences. However, the universal theoretical background provides helpful clues that help ordering empirical facts, and suggesting types of experiments that promise to be most fruitful for a better governance of the dynamic processes that characterize systemic risks. Such an approach may even pave the way for quantitative methods of mathematical modeling for an in-depth study of the system's internal mechanisms.

2.1. Elementary Processes at the Microlevel

Looking from an overarching perspective across all domains, systemic risks are macroscopic phenomena of dynamic structure generation resulting from actions of agents on the microlevel. In physics and chemistry, such phenomena have been studied in detail on the basis of solid scientific and mathematical grounds and have generated insight into the elementary processes on the microlevel. Prototypes of such studies in physics are the theory of synergetics developed by a generalization of the theory of laser light^[8] and the theory of dissipative structures as emerged from nonequilibrium thermodynamics, in particular, with application to the generation of chemical structures.^[9–11]

We learn from those studies that dynamic structure generation in the systems of physics and chemistry require first open systems capable of exchange of materials and energy with the environment and second system complexity in the sense of the elements being capable of feedback interactions resulting in circular causality. The characteristics of complexity are thus strong nonlinear interactions between the relevant agents, such as photons and atoms in a laser or molecules capable of autocatalytic reactions in a chemical system, in combination with external impact quantified by suitable control parameters. Such control parameters of the referred-to model systems are the pumping energy in a laser or the influx of chemicals in appropriate reactors. When the control parameters exceed threshold values, the system is brought into a state of instability. In such states, the system tests its various intrinsic modes of macroscopic behavior, such as the various forms of light waves in a laser or of chemical patterns in an autocatalytic reactor. A stability analysis then reveals that most modes behave conservatively in the sense that they respond with small changes to small changes of perturbations, without any macroscopic change of the system state. However, some modes, in competition with the rest, grow exponentially to amplitudes which react back to the collective behavior of the system elements and forces the system into a new and unforeseen state.

Original states thus collapse in the neighborhood of points of instability. New ordered states organize themselves on the basis of circular causality between the microelements and the macrolevel. Thus, in a laser, a particular monochromatic light wave emerges out of the many possible light waves of standard lamp light by competition as the new structure. In a chemical reactor, particular patterns emerge from an originally homogeneous distribution of chemicals, such as rings or stripes or also regular dynamic changes of patterns such as a chemical clock. These modes winning the competition are referred to as order parameters, describing the new macroscopic order in the system, such as static or, in particular, dynamic structures and patterns. So, in a laser, the particular optical wave would be an order parameter, while in chemical structure generation it would be the emerging dissipative structure. The macrodynamics of the system is formulated in terms of the order parameters, which for the simple model systems can be cast into mathematical terms.

A considerable decomplexification as compared to the microlevel is associated with the formulation of the system's behavior in terms of order parameters. They are real and thus measurable macroscopic phenomena, not just mathematical concepts, and their number is much smaller than those that determine the state of the system on the microlevel. Due to this information compression, the dynamics of the order parameters agrees for widely different systems with otherwise no relationship between them whatsoever. This is the formal reason for many analogies in the basic patterns of dynamic behavior in the various domains, referred to here as homomorphism. The dynamics on the microlevel and on the macrolevel proceed on entirely different time scales, in the sense that order parameters change much slower than the rapidly changing and rather individual microstates. The system-specific microdynamics may then be considered as fluctuations which relax to their equilibrium values almost immediately and thus can be neglected in the analysis of the macroscopic dynamics of the system.

2.2. Empirical Evidence for Homomorphism

The model systems of physics and chemistry, although complex, are controlled by relatively simple and well-understood interaction laws. Clearly, in the systems where systemic risks are relevant for the society quite different and much more complicated interaction laws are in effect, with nonlocal, adaptive decision interactions including memory effects in human societies as the most complex situation.

Fortunately, there is overwhelming empirical evidence that the fundamental patterns of dynamic behavior do not depend crucially on details of the agent's interactions. This has the remarkable effect that systems with very different types of agents show rather similar patterns of dynamical behavior. We refer to this phenomenon as homomorphism. The mathematical origin of this empirical fact is identified by the study of the elementary processes on the microlevel of the model systems. It is shown there that the dynamics of systems can be quite generally formulated in terms of order parameters with rather universal dynamics. While these order parameters in the general case do not arise out of rigorous theory, they can be established from empirical observation

of a system. In particular, this homomorphism paves the way for a universal mathematical formulation of systemic risks where the individualities of the systems are taken care of by specific formulations of the microdynamics.

Most fundamentally, dynamic structures, associated with systemic risks in all domains, are phenomena of emergence, typically out of system instability. In all domains, they are collective effects resulting from the elementary actions of the agents at the microlevel. So, they are only observable at the macroscopic level, not at the microlevel of the agents. Still, the macroscopic homomorphism invites looking for homomorphic mechanisms of structure generation even at the microlevel. Even in socioeconomic systems where the agent's interactions are clearly entirely different from those in the simple model systems, the basic mechanisms of dynamic structure generation and pattern formation are pretty much analogous. Here also strong and reinforcing interactions generate feedback loops and circular causality between the macroscopic structures and the actions of the agents. Agents learn and adapt to their environment created by them. It is frequently the interaction of human individuals as agents with a field of information from the media, public opinion, or other sources which stabilizes the structures in socioeconomic systems. So, local patterns are again able to spread over the whole system by appropriate information distribution.

Another general phenomenon of dynamic structure generation observable in all domains is that of tipping and cascading. In the model systems, it has been established that boundary conditions keeping the system away from the dead state of static equilibrium and exceeding threshold values may drive a system into a regime of instability out of which new dynamic structures may suddenly emerge, such as a laser light wave or a chemical structure. Similar patterns are found in the study of pandemics and the spread of disease.^[12] Further examples are failures of infrastructure, such as the electrical grid,^[13] the break-down of a financial system,^[14] or such phenomena as public opinion formation or economic innovations. Like in the model systems, there are parameters indicating a region of instability. Generally, such parameters are nondimensional in nature balancing enhancing and hindering effects by a suitable combination of external and internal quantities, such as the ratio of local uprisings to police interventions in the forefront of revolutions^[15] or the size of the economy to the amount of private debt in the onset of a financial crisis^[16] or the index of conflict-related news before the outbreak of war.^[17]

Finally, it is observable in all domains that the behavior of the system exhibits an individual history, consisting of an interplay between chance and determinism. Examples from socioeconomic systems are traffic congestion, political revolution, economic competition, or urban settlements. So, the adoption of one of the alternative technologies within a society or the market success of a particular company can be greatly influenced by minor contingencies about who chooses which technology or which company at an early stage, which clearly reminds of the butterfly effect usually observed in chaotic systems. This early choice determines the further fate of the system and explains the remarkable success of one technology or one company over others in the end. Such a path dependence excludes any purely local and momentary origins of systemic risks, for example, such of a stock market crash^[18] or the refugee crisis.^[19] It is indispensable to study the

history of the system if adequate understanding of the dynamics of the system is aimed at.

2.3. Systemic Risks as Phenomena of Complexity

Like the model systems of physics and chemistry, the dynamics and thus the associated systemic risks in any system can only be properly understood and analyzed when considered in the framework of complexity science. It then becomes generally clear that dynamic processes do not develop by some sort of central control and programming but rather emerge as a result of nonlinear feedback interactions between the agents of a system and its environment. Traditional concepts of causality and equilibrium are incompatible with empirical observation and systems behave often counter-intuitively. The complex system approach realizes that the emergence of systemic risks in all domains, notably those in ecology and with human agents, is not to be understood as the sum of single intentions, but the collective result of nonlinear interactions. A common feature of all nonlinear systems is the emergence of collective order phenomena. The dynamics is formulated in terms of order parameters and understood as a sequence of phase transitions of a system exchanging material, energy, and information with its environment. There is plenty of evidence to prove the necessity to analyze systemic risks in the framework of complexity science.

A relatively simple but quite instructive example can be taken from sociobiology and it is that of state-forming insects, such as ants. In the beginning the state-forming process makes the impression of an uncoordinated phase of microscopic activity. However, as first elements of macroscopic structures become noticeable, the system evolves to an inhomogeneous stationary state. In this phase, ant colonies seem to form a deterministic system, each insect following rules of behavior which are centrally controlled, with the result of a remarkably efficient transportation macrostructure of the colony. A closer look, however, reveals that this structure is the result of many chaotic fluctuations with selection processes among the many modes of macroscopic transportation behavior resulting in a seemingly stable macroscopic structure. The associated order parameter is a particular path network from a resource of food to the nest, associated with a particular location of the food resource, and representing a network of signals between the ants. Thus, this phenomenon of swarm intelligence is based on the interactions between the ants and at the same time acts back upon them in circular causality. Again, we identify the typical feedback between the micro- and the macrolevel of a complex system as the basis for self-organization and emergence of order. Observation over some time then makes it clear that the macrostructure may change spontaneously, similar to the phase transitions discussed above when some fluctuation induces such a change. Such a fluctuation may be the discovery of a new food source. Then, the existing transportation network may become unstable and a new transportation network may establish itself in phase transition. If the new food source has been overestimated, there is the systemic risk of disturbing the colony which finally will lead to the parallel existence of two transportation systems with the associated weakening of the coordinated actions of the ants. There is no simple

causality, not to speak of an equilibrium state observable in an ant colony.

A further example is readily available from economics. The traditional assumption of free market economy is that the interactions of single micro-interests achieve the common macroeffect of welfare. In this view, economy is a complex system of many competing microinterests. The dynamics of their interactions in a self-organizing process leads to a final state of equilibrium between supply and demand. Here, individual human behavior is assumed to be rational, regular, and predictable, leading to a linear economic model. Such linear models do not take into consideration unpredictable irrational behavior, restrictions imposed by the environment, and nonlinear, nonadditive interdependencies between individuals and their actions. In reality, the system of a free market economy is an open system in permanent exchange of matter, energy, and information with other markets and nature. Thus, endogenous nonlinear systems with impressed waves of exogenous forces are more realistic models of economics. An economic system will, therefore, not approach any type of equilibrium state but rather undergo processes of dynamic structure generation, triggered by even minor changes in the boundary conditions as control parameters. Policy measures as impacts on an oscillating system may cause effects opposite to those intended up to driving the system into the chaotic regime. The agents of a market system are humans capable of learning and adapting to new situations, including memory effects. Sensitive reactions in the system due to short time fluctuations of consumer preferences, uncoordinated dynamics of the production side, speculations on markets of commodities, and real estate are common experience. Even fluctuations on a small scale, such as technical innovations, can organize themselves by the mechanism of selection between stable and unstable modes of the system to large scale growth phenomena and may build up to entirely new macroscopic states of the system. While indispensable for economic welfare, such mechanisms may well lead to systemic risks and even disasters like stock market crashes and spread of unemployment with associated mass impoverishment. As a consequence, there have been formulated ideologies which suggest to abolish the dynamic structure generation by self-organization in economic markets altogether and replace it by a central distribution machinery, that is, turn to the artificial linear equilibrium theory. This, of course, can only be sensible in closed systems as established theoretically as well as practically. In open systems, such as a modern economic system, historical experience has impressive facts about the nonworkability of such an approach.

As a final example for systemic risks as phenomena of complexity, we consider the emergence and dynamics of divided societies as a process of dynamic structure generation. This process is relevant in social unrest, in revolutions, and in changes of political systems. The system to be considered may be a state with well-defined borders. Across the borders, external control parameters impact upon the system such as a transfer of people and information. The agents in the system are manifold, at the lowest level the humans in the system, but at higher levels autonomous substructures formed by them such as political parties, mass media, and the government. The emergence of a divided society is invariably associated with the polarization and radicalization of the public opinion. Processes at the microlevel of the agents gen-

erate a predominant public opinion in a bottom-up outcome of a macrostructure. This macrostructure acts back down upon the agents of the system in circular causality. The order parameter is thus the public opinion itself, for example, the distribution of approval, refusal, or indifference with respect to a particular political issue, such as that of an upper limit of refugees to be accepted in a country during one year. The goal of the study is to scrutinize under which circumstances sudden changes in public opinion can set in, that is, a phase transition, and so, as usual, the time dynamics of the order parameter. Such macroscopic dynamics is triggered by the microlevel processes, in particular changes in political opinions of the citizens. Such changes result from interactions of the citizens among each other as well as with the mass media, the collective opinion macrostructure, and government propaganda. In the beginning of phase transitions, a destabilization of the system will be observed. External control parameters inducing such instability may be factual or even only assumed boundlessness of migrants from culturally incompatible societies or the threat of economic drop by global effects, for example, by political turnabout in some important interacting country. Besides such external control parameters, there are internal properties of the system inducing nonlinear interactions promoting instability, which here may be spiritual bondage, social unequalness, and loss of trust in justice and safety. In a situation of instability, a considerable decomplexification becomes noticeable. This invites to look for simple measurable parameters. So, a situation of instability on the eve of societal unrest makes itself noticeable by an accumulation of extraordinary events, such as unapproved demonstrations, terror attacks from the inside of the society, and aggressions against fringe groups of the society. In such situations of instability, a dramatic sensitivity of the macrostructure of the system with respect to accidental fluctuations in the microstructure presents itself, such as crimes attributed to single migrants, fake news about foreign infiltration resulting in the appearance of a new political party, or other normally ignored events. Close monitoring of such phenomena helps to recognize the onset of a phase transition, here a sudden turnover of public opinion with the consequence of the emergence of a divided society.

3. Mathematical Modeling

Looking at systemic risks on the basis of complexity theory indicates the nature of phenomena which are to be expected in the dynamic evolution and the associated systemic risks. So, one is advised to be alert to different macroscopic time scales: while the system may evolve slowly and hardly noticeably under the continuous influence of some exogenous impact; there may at one moment appear a sudden tipping, a harmful disaster, of unpredictable consequences. One is further motivated to look for characteristic parameters indicating instability regions, which will announce themselves empirically by irregularities, as discovered, for example, by a time series analysis. Finally, historical insight into a system should be valued since the path the system takes cannot be understood without a historical perspective. In this way, complexity science provides a mental framework for ordering the analysis and helps to systematize the empirical considerations of systemic risks in socioeconomic systems.

Beyond this remarkable value of the general framework, the mathematical instruments of complexity science can be applied to the analysis of systemic risks, with the goal of getting quantitative insight into the systemic risks to be expected in a system. The three main approaches are the formulation in terms of differential and difference equations, agent-based simulation, and complex network theory.

3.1. Differential and Difference Equations

The most natural mathematical formulation of dynamic systems is based on differential and difference equations. It turns out that rather different mathematical models tailored to rather different systems essentially reproduce the same fundamental patterns of dynamic behavior as a consequence of the homomorphism discussed above. So, contrary to many standard problems of physics and chemistry, an individual mathematical model in complexity theory in the form of an iterated function or a differential equation is just the beginning of an understanding, whereas its evaluation over time produces universal and unforeseen dynamical structures. Well-defined rules, often surprisingly simple, when applied in active repetition without any intentionality over and over again, lead to an evolution in time showing a remarkable creativity and richness of structures, not to be expected from the simple underlying model specifying the rules.

3.1.1. Top-Down Models

One branch of models based on differential equations describes the dynamics of systems in terms of formal parameters without any explicit reference to the interactions of the agents. From a methodological perspective, this is a top-down approach to the generation of a dynamic model. A mathematical model is assumed and the dynamic behavior of the associated system is deduced by solving the differential equation under suitable initial and specifying conditions. A paradigmatic example is the logistic map which illustrates the basic concepts of complex dynamical systems.^[20] Mathematically, this difference equation is defined by a quadratic and thus nonlinear recursive formula mapping the value of a function onto itself depending on a control parameter. Its continuous version is the logistic curve or differential equation. For different values of the control parameter, all typical types of dynamical behavior can be modeled, from an equilibrium fixed point attractor at low values, over a limit cycle at increasing values where the system jumps periodically between two states up to the phenomenon of chaos, that is, an entirely irregular dynamic behavior depending sensitively on the initial conditions. It is far from self-evident that a simple equation with simple rules is capable of reproducing such a richness of dynamic structures. It clearly shows that complex dynamic behavior does not require complex rules. There is a wide spectrum of applications of the logistic model, from chemical reaction rates over growth of animal and plant populations up to the growth of towns.^[21] Quite similar patterns are obtained from the Lotka–Volterra equations,^[22] which find extensive use in biology, ecology, and epidemiology. In particular, they have been used to investigate the relationship between predator and prey and the associated systemic risks with

respect to the stock of a population. The fact that different equations lead to the same basic patterns underlines mathematically the homomorphism of dynamic structures in different domains, as discussed in empirical observation above.

3.1.2. Bottom-Up Models

In tailoring the model to a particular application, it could be adjusted to the problem under consideration from a set of data corresponding to time-dependent events of the dynamic system. This would be a bottom-up approach to model building. However, in most practical cases associated with systemic risks, notably in socioeconomic systems, measurements of dynamic events will not be available in sufficient quantity or quality to proceed along these lines. Then it is more practical to design a bottom-up model on the basis of plausible assumptions about the microprocesses of the agents. Agents make decisions, that is, they choose one of several behavioral alternatives in conflict situations with particular probabilities. This then leads to the formulation of so-called master equations^[23]. These are differential equations formulating the time evolution of a distribution of order parameters of a complex system in terms of probabilistic actions of the agents.

In the social sciences, this approach has been worked out for numerous applications and the associated field is referred to as sociodynamics.^[24] The general machinery of this approach is a two-step procedure in mutual feedback, starting from a thorough empirical analysis of the system in the framework of complexity science and casting this analysis into a mathematical formulation. Examples for systemic risks treated quantitatively by this approach are the dynamic structure generation in migration of interacting populations^[25] or the emergence of divided societies. We briefly discuss these systems here for illustration.

Starting with the empirical analysis, it is first recognized that the agents in such applications are obviously the individuals of the society. The analytical goal is to scrutinize the conditions under which an originally homogeneous distribution of individuals over different areas (migration) or over various public opinions (divided societies) in the system may become unstable and perform a transition to nonhomogeneous distributions. Hence, the parameters describing the distribution of individuals over the areas or the opinions represent the appropriate order parameters. The microlevel of individual decisions is distinguished from the macrolevel of the dynamic collective processes. The macroprocesses result from the actions of the agents on the microlevel. Since there is no rigorous theory for these actions, contrary to the model systems of physics and chemistry, assumptions of a probabilistic nature are made for them which naturally leads to a probability distribution of the order parameters. The time-dependent differential equation for this probability function is the master equation. The interactions in this approach are not direct in the sense that they are between two or more agents. Rather, they are decisions which induce behavioral changes of the total system. Such indirect interactions are especially relevant for systems where the dynamics is induced by the socioeconomic field, such as represented by the media, public opinions, norms, trends, and the environment, which influence the decision preferences of the individuals. Differences between human and nonhuman complex systems become very clear in this modeling approach.

At the microscopic level, human actions are intentional, for example, guided by consideration of utilities, and nonlinear, that is, dependent on individual as well as collective interactions. These interactions represent internal driving forces. Structures initiated by actions of the agents will act back upon them and thus will stabilize the structures by circular causality. As a prerequisite for structure generation to happen, the system must have been brought into an unstable condition. This can be achieved in principle either by external impacts, measured in terms of external control parameters exceeding a threshold, or by internal effects, passing a critical threshold of internal control parameters. Normally, it is a critical combination of both effects. External impacts may be political regulations setting rules for the living together of individuals in the society or also the informational flow into the system from other societies. Internal effects may be the advent of strong cultural or ethnic feelings leading to agglomeration or segregation movements in combination with a general and global positive inclination to radicalism. When a regime of instability is reached by a critical combination of external and internal impacts, random fluctuations become important and may drive the system out of the homogeneous and into a heterogeneous distribution as a new dynamic structure. Such random fluctuations may be of political or economic origin, such as external political impacts or internal economical perturbations with which some parts of the population are charged. In detail, like in the physico-chemical model systems, fluctuations in the form of chaotic and uncoordinated elementary dynamic interactions lead the system to test its various different modes of potential macroscopic behavior and finally select one or a few. This leads again to the appearance of macroscopic order parameters in terms of which the dynamic structure generation can be described. In the present case, these order parameters are the number of individuals of particular groups distributed over the various areas or opinions in the system. This is very much less than to be expected from the multitude of agents and their elementary interactions. Most of these microscopic variables turn out to relax rapidly to their equilibrium values and just can be eliminated from the dynamic model.

This empirical analysis can be cast into a mathematical formulation. For this purpose, it will be required to formalize the elementary dynamics in terms of probabilities, since no deterministic laws will generally be known. In the master equation, the actions of the agents are indirect in the sense that they do not interact with each other directly but rather change the macroconfiguration of the system. The elementary agent actions thus consist of transitions of one individual from one region or political opinion to another thus changing the macroconfiguration directly. This transition is formulated in terms of a particular probabilistic transition rate reflecting the attractiveness of the transition step, referred to as a utility function. The utility functions contain empirical parameters such as those measuring the attractiveness of a transition. They also contain the original as well as the new macroconfiguration and thus formulate the crucial phenomenon of circular causality between the agent's actions and the macroscopic structure. This implies that neither the migration nor the public opinion formation can be explained by the free will of single persons but rather is influenced by the social environment. Balancing the transitions rates in and out of a particular macroconfiguration leads to a stochastic evolution model, a differential equation referred to as the master equation, which

has to be solved numerically. The final result of the quantitative model is then information about the probability of a certain evolution of the system. After proper averaging, an evolution phase portrait of the order variables can be obtained. It should be noted that the master equation has a universal form for all applications. Its individuality is contained in the probabilistic transition rates, in which all empirical knowledge is incorporated. Since, contrary to the model systems of physics and chemistry, no rigorous formulation is available for the microdynamics, the master equation formulation in socioeconomic applications has to rely on empirical assumptions about the agent's actions. So, any prediction of the future behavior of the system will not be possible. Instead, scenarios can be studied which shed light on the relevance of certain agent actions for the macroscopic system dynamics. This detailed information represents a considerable progress with respect to quantifying systemic risks in comparison to a purely empirical analysis.

3.2. Agent-Based Simulation

Master equations are not easy to solve for complex applications. So, the complexity of scientific models is limited by mathematical tractability. An alternative approach is to abstain from a closed mathematical formulation altogether and instead turn to a computer simulation of the agent's actions and study the resulting emergence of macroscopic structures. Contrary to the master equation formalism consideration is turned to direct interactions between the agents, for example, opinion changes caused by discussions between individuals instead of feedback by some sociocultural field. The agents are unique and autonomous entities that interact with each other and their environment locally. As autonomous entities agents act independently of each other and pursue their own objectives. They show an adaptive behavior in the sense that they adjust to the current state of themselves, of other agents, and of their environment. Agent-based models cross the various levels of a system. They study what happens to the system on the basis of the agent's actions but, in circular causality, equally well what happens to the agents due to the macroscopic dynamics of the system. This represents a particular thorough insight into the origins of systemic risks and their assessment and control.

The modeling machinery is analogous to that for the master equation approach, that is, a two-step procedure of empirical analysis of the system and incorporating this into a simulation computer code. So, we may renounce illustrating the procedure by a particular example and instead point to the vast literature on application of agent-based simulation to the study of the dynamics of complex systems.^[26–28] Clearly, in quantitative studies for socioeconomic systems, some of the same basic limitations apply as in the master equation formalism, although the approach is more versatile. There remains the problem of a lack of fundamental theory for the properties and the actions of the agents. They must be modeled on the basis of assumptions with empirical parameters. So again, only seldom will enough data be available to allocate meaningful values for the model parameters and most studies that have been made have the character of scenarios.

We finally note that agent-based simulations are also common practice in the physicochemical model world. In fact, some

technical tricks in performing these simulations have been inspired by those earlier applications, notably molecular dynamics and Monte Carlo studies for the properties of fluids.^[29,30] It has been learnt there that such computer simulations do not have their primary value in predicting the system's macroscopic behavior. Much rather they are tools for developing theories of a system's behavior on the basis of the microprocesses or, alternatively, as tools to gain insight into suitable interaction rules between the agents to generate a desired macroscopic behavior of the system. In applications to systemic risks, this insight in the consequences of modifying the interaction rules between the agents is a sensible route to information about successful governance strategies.

3.3. Complex Network Theory

More often than not, systemic risks appear in complex systems with a distinct network structure. Typical examples from technology are energy supply grids, urban freshwater supply, transport, and information/communication networks. These systems are extremely vulnerable^[31] and thus, due to their importance for society life and welfare, represent a significant sector in the study of systemic risks. Beyond these technical infrastructures, also systemic risks in financial systems are best understood when considered as a network structure and this frequently also applies to social systems. So, complex network analysis recommends itself as a further mathematical tool for the analysis of system risks.

Beyond the mathematical modeling approaches discussed above, complex network theory opens the chance to generate new insights into network growth mechanisms and interdependencies among network partners shedding additional light on points and causes for systemic risks. Since networks are ubiquitously present in almost all systems in biology, ecology, technology, and sociology, there is a considerable amount of empirical knowledge about controlling such structures that may well be transferred to any domain in the sense of homomorphism and be made fruitful in managing systemic risks. Information may thus be obtained from rather different empirical sources about how the underlying network topology influences the system behavior and its inherent risk characteristics. Just as the topology of a power grid affects its resilience to failure and breakdown, analogous mechanisms are in operation in financial as well as in social networks, and much can be learnt from these analogies. Quite generally, complex network theory comprises the study of the topology of the network graph and its relationship to its vulnerability to systemic risks and the study of its dynamic behavior through functional models based on the communication processes within the network responsible for the emergence of cascading of failures taking place in it.

The topological analysis aims at capturing the structural properties of a network. For this purpose, the network is modeled as a graph whose nodes represent the system units and the links model the interactions between directly connected units. The actual structure of the network of interconnections between the units is a critical feature of the system. Redundant wiring, in technical infrastructures, is known to promote their stability and robustness. A crucial aspect of complex network analysis is the interplay between the network structural characteristics and its

dynamic performance. Models have been developed which allow analyzing the system response to cascading failures and can be used to guide a successive detailed simulation concentrating on the most relevant processes and network components. Clearly, such an analysis, if it is to be a meaningful prediction tool in a practical situation, has to rely on a large amount of data. In technical infrastructures, such data would concern the technical characteristics of links and nodes, load requirements, failure probabilities etc., with similar information requirements for other types of networks, such as in financial or social systems. This requirement of empirical information is ubiquitous in all applications of mathematical modeling and is the primary limitation to reliable predictions. However, here as in the mathematical modeling approaches discussed before, valuable insights may be obtained from parameter variations and associated scenario studies.

An example of topological and weighted analysis is available for the Swiss 220 kV/380 kV high voltage transmission system, made up of 161 nodes connected by 219 overhead lines.^[32] The analysis is based on the assumption of a constant and universal annual failure rate per km for the whole network, which is the number of failures that occur in 1 year along 1 km of overhead line connecting two nodes. The analysis identifies the most vulnerable lines of the network, as a first basis for the systemic risk assessment. In order to capture the dynamic behavior of the system, a failure cascade modeling has to be performed. Results of such a study are available for the 380 kV Italian power transmission network.^[33] In the model, the components of the grid are assumed to have a load limitation beyond which they fail. To study a possible cascade of failures spreading over the whole network an initial perturbation is introduced imposing on each component an additional load. A component will fail when the sum of the initial and the perturbation load exceeds the limitation load. Then an additional load is transferred to each of the systems components. As the total load of the grid increases, a cascade of failures becomes more likely. Each failure of a component causes a redistribution of power flow in the network imposing stress on the other components which may lead to a blackout. This does not normally happen gradually or uniformly but rather as an event of phase transition in an unstable situation of the grid. The dynamic phenomena in power grids thus again show the typical patterns of emerging dynamic structures in open complex systems.

A major example of the relevance of complex networks is given by the financial crisis that hit the world economy in 2007 and the following years. It has already generated a substantial literature on systemic risks in the domain of finance. Whether the insights available so far will be sufficient to avoid another crisis of potentially even larger proportions remains to be seen. We have discussed the global financial crisis as a paradigmatic example of systemic risks in ref. [5].

4. Conclusions

As the modern world savors its advances, it grows ever more fearful of the potential threats that accompany them. Systemic risks are now a troubling addition to the already sizable repertoire of conventional risks. They are real, they appear again and again, as headlines show, they present intractable challenges, and they demand new forms of governance.

The first step toward an effective governance approach is the need to understand systemic risks and their implications. Until this day, we lack an adequate understanding of the structure and dynamics of systemic risks. The lack of a well-defined event space and sufficiently defined preferences impede the application of conventional risk assessment methods based on the combination of probability distributions and the extent of damage.

The focus should rather be on multiagent models that link the microlevel to the macrolevel and include emerging properties, since each agent is linked to other agents by multiple feedback loops. The experiences from the physical and chemical sciences can be used as a heuristic tool for building such models and fill them with substantive empirical data. The challenge will be to improve our modeling capability to include intentional behavior in models considering large degrees of freedom and variability. In addition, systemic risk evolves dynamically and produces behavioral changes over time in a historic time dependency. It is still unclear how much of these dynamics is idiosyncratic and how much generalizable. The concepts that were laid out in this Essay may serve as a guidebook for collecting empirical data and constructing complex multiagent models in an effort toward a more profound analysis and toward governance strategies across domains.

Furthermore, management concepts that have been developed to understand conventional risks are not sufficient for dealing with systemic risks because they are often too reductionist and limited in scope to account for complex system interactions and challenges. As such, managing systemic risks requires a more comprehensive approach to hazard and impact identification, risk assessment, and risk management. The options for managing complex systems in a targeted manner (e.g., with mitigation) are limited, and interventions can have unexpected and uncontrollable consequences which may look as stochastic and chaotic. The main lesson learned from this Essay is that homomorphic patterns of complex systems could be an important heuristic tool to improve our understanding of complex risks and their characteristics.

Acknowledgements

Financial support by the Berlin–Brandenburg Academy of Sciences, Berlin (BBAW), and the Institute for Advanced Sustainability Studies, Potsdam (IASS) as well as discussions in the working groups on systemic risks of BBAW and IASS are gratefully acknowledged. Note: The keywords were updated on November 5, 2018 after initial online publication.

Conflict of Interest

The authors declare no conflict of interest.

Keywords

complexity analysis for social phenomena, homomorphism, systemic risks, systems science simulation, mathematical models

Received: April 8, 2018

Revised: June 12, 2018

Published online: August 27, 2018

- [1] E. A. Rosa, O. Renn, A. M. McCright, *The Risk Society Revisited: Social Theory and Governance*, Temple University Press, Philadelphia **2014**.
- [2] O. Renn, *Das Risikoparadox: Warum, wir uns vor dem Falschen fürchten*, Fischer, Frankfurt/Main **2014** p. 102.
- [3] OECD, *Emerging Risks in the 21st Century: An Agenda for Action*, OECD, Paris **2003**, <https://tinyurl.com/kcd94ar>.
- [4] G. Kaufman, K. E. Scott, *The Indep. Rev.* **2003**, 7, 371.
- [5] O. Renn, K. Lucas, A. Haas, C. Jaeger, *J. Risk Res.* **2017**, <https://doi.org/10.1080/13669877.2017.1409252>.
- [6] O. Renn, *Environ. Sci. Policy Sustainable Dev.* **2016**, 58, 26.
- [7] R. Schröter, A. Jovanovic, O. Renn, *Plant@Risk* **2014**, 2, 125.
- [8] H. Haken, *Synergetics*, Springer, Berlin **1977**.
- [9] I. Prigogine, R. Lefever, *J. Chem. Phys.* **1968**, 48, 1695.
- [10] I. Prigogine, *From Being to Becoming*, W.H. Freeman, San Francisco **1980**.
- [11] P. Gray, K. S. Scott, *Chemical Oscillations and Instabilities*, Oxford Press, Clarendon **1990**.
- [12] L. Chen, F. Ghanbarnejad, D. Brockmann, **2016**, arXiv.org.
- [13] A. M. Koonce, G. E. Apostolakis, B. K. Cook, *Int. J. Electr. Power Energ. Systems* **2008**, 30, 169.
- [14] R. Peckham, *Econ. Soc.* **2013**, 42, 226.
- [15] R. Schröter, A. Jovanovic, O. Renn, *Plant@Risk* **2014**, 2, 125.
- [16] H. Minsky, *Stabilizing an Unstable Economy*, McGraw-Hill Professional, New York **2008**.
- [17] T. Chadeaux, *J. Peace Res.* **2014**, 51, 5.
- [18] D. Sornette, *Why Stock Markets Crash*, Princeton University Press, Princeton **2003**.
- [19] K. Lucas, *Denn wir wissen nicht, was wir tun*, VDI-Nachrichten, May 17, **2016**, No. 21.
- [20] P. Collet, J. P. Eckmann, *Iterated Maps of the Interval as Dynamical Systems*, Birkhäuser, Basel, Switzerland **1980**.
- [21] G. F. Mulligan, *Geogr. Anal.* **2006**, 38, 344.
- [22] A. J. Lotka, *Analytical Theory of Biological Populations*, Plenum Press, New York **1998**.
- [23] H. Haug, *Statistische Physik-Gleichgewichtstheorie und Kinetik*, Springer, **2006**.
- [24] W. Weidlich, *Sociodynamics*, Dover, New York **2000**.
- [25] W. Weidlich, G. Haag, *Collect. Phenom.* **1980**, 3, 89.
- [26] S. F. Railsback, V. Grimm, *Agent-Based and Individual-Based Modeling: A Practical Introduction*, Princeton University Press, Princeton **2011**.
- [27] N. Gilbert, *Computational Social Science: Agent-based social simulation*. In: Agent-based Modelling and Simulation. Bardwell, Oxford, **2007** pp. 115–134. ISBN 978-1-905622-01-6.
- [28] D. Helbing, *Automation of Society is Next: How to Survive in the Digital Revolution*, Createspace, North Charleston, SC **2015**.
- [29] K. Lucas, *Molecular Models for Fluids*, Cambridge University Press, New York **2007**.
- [30] B. J. Alder, T. E. Wainwright, *J. Chem. Phys.* **1957**, 27, 1208.
- [31] W. Kröger, E. Zio, *Vulnerable Systems*, Springer, London **2011**.
- [32] E. Zio, G. Sansavini, *Risk Anal.* **2011**, <https://doi.org/10.1111/j.1539-6924.2011.01584.x>.
- [33] E. Zio, G. Sansavini, *IEEE Trans. Reliab.* **2011**, 60, 94.