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Green Growth Mechanics:

The Building Blocks

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Abstract Green growth rhetoric has become widespread in policy circles. It holds the promise that solving environmental problems need not harm the economy; quite the opposite, it is said to offer new economic opportunities. Yet, until now there is little analytically sound work on the possibility of such a dynamics. We investigate conditions under which a transition from "brown" to "green" growth can improve the economic situation both of present and future generations. We show that it is misleading to represent the situation as a prisoner's dilemma, where everybody is tempted to free ride rather than cooperate. It is more appropriate to describe it with the metaphor of a stag hunt, a situation where joint action can achieve a Pareto improvement not threatened by the so-called tragedy of the commons. This is due to the combination of three well-documented phenomena: the fact that a major aspect of technical change is learning by doing, the fact that learning by doing can develop in different directions, and the indeterminacy of labour markets resulting from the difficulty of matching the skills of people with the demand of firms. Each one of these three aspects is relevant for green growth policies, but it seems that only by combining them such policies can be successful.

Keywords Green Growth · Multiple Equilibria · Optimal Growth Models · Game Theory

1 Introduction

The term "green growth" is used in many, often quite vague ways (e.g., Rische et al 2014). The key idea is to modify the growth path of the world economy so as to avoid environmental disruption while achieving the economic growth needed to overcome poverty worldwide. Most research about this general idea has been performed in view of climate change (see, e.g., Intergovernmental Panel on Climate Change 2014), and the main result is that it is definitely possible to reduce greenhouse gas emissions to avoid major climate change, if one is willing to accept non-trivial but technically feasible reductions in economic growth. The political pressure to avoid such growth reductions, however, seems hard to overcome at global scale. The question then arises whether it is also possible to massively reduce global emissions while enhancing global growth. This idea has gained increasing traction in public debate and also among some highly influential decision-makers (e.g., Apple et al 2017). But so far, there is no thorough analytical basis for this version of the green growth idea. The purpose of the present paper is to provide such a basis.

Against this background, for the purposes of the present paper we define green growth against a benchmark of brown growth, i.e. a plausible growth path of the world economy in the 21st century. As green growth we then define a different growth path where at any

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point in time GDP growth is greater and greenhouse gas emissions are lower than along the brown path. The aim of this paper is to clarify whether there are plausible conditions under which green growth in this sense is possible.

Conditions are plausible if they are compatible with long-term empirical patterns captured by state-of-the-art work on economic dynamics. In particular, it is not plausible that developing and reliably implementing zero-emissions technologies worldwide in a few decades can be achieved without massive investment of economic resources. Nor is it plausible that green growth can be achieved without major technological innovations – certainly to bring costs down in many areas, e.g. for energy storage without hydrocarbons. Similarly, it is plausible that green growth will most likely require the development of whole new socio-technical systems, e.g. for mobility.

Commonly used climate policy models by construction find a long period of substantial net costs for reducing greenhouse gas emissions, before eventually the expected large-scale benefits from such reductions will materialise (Wolf et al 2016). In a nutshell, climate policy has to deal with three time lags: first, it takes years and decades to reduce emissions; second, it takes further decades for emission reductions to make a significant difference for the concentration of greenhouse gases in the atmosphere; third, it takes decades and even centuries for reductions of greenhouse gas concentrations to translate into reduced impacts of climate change. At the same time, however, these models assume that reducing an input like fossil fuels to the economy will lead to costs that cannot be compensated by increasing some other input. Thus, green growth scenarios are excluded from the set of possible model outcomes in the first place. Green growth becomes a narrative without an economic “mechanism” that can be made plausible using a set of equations.

When it comes to describing the possibility of green growth, the difficulty economic modelling is faced with is the problem of multiple equilibria and equilibrium selection: the idea of having at least two possible stable growth paths (the “brown” business as usual and a green growth path) means that there are at least two equilibria of the economic system. In the canonical framework of general equilibrium theory (Arrow and Debreu 1954) multiple equilibria are the norm, and single equilibrium systems an exception (Sonnenschein 1972; Mantel 1974; Debreu 1974). However, the canonical framework does not make any statement about equilibrium selection or the out-of-equilibrium dynamics of the system. In fact, these dynamics can be quite arbitrary (Saari 1995). In practical modelling, these difficulties have been discarded by various assumptions (e.g. a single representative consumer and strictly convex production possibility frontiers) that exclude multiple equilibria and hence implicitly exclude the possibility of green growth.

Furthermore, the idea of green growth suggests the possibility of a win-win situation, which means that some equilibria are Pareto-superior to others. But then the question arises how an economy of – as usually assumed – rational and well-informed agents and functioning markets could possibly end up in a Pareto-inferior equilibrium.

These problems are not merely of theoretical interest. Given the global age structure, out of a world population of about 7.5 billion people, at least four billion are able to join the active workforce of the global economy. Taking account of widespread underemployment, this workforce is in fact smaller than three billion (for relevant data see International Labour Organisation and International Institute for Labour Studies 2013) and there can be little doubt that the world economy is far from a Pareto optimum.[1]

The situation of developing countries is only part of the problem. At the technological frontier of the world economy (Caselli 2016), the global financial crisis was a major experience of Pareto sub-optimality. Moreover, and possibly related to the financial crisis, investment rates and productivity growth have decreased over the past decades, while income inequality has increased. These trends seem to continue, raising concerns about new forms of secular stagnation at the technological frontier itself (Summers 2013).

[1] It is worth noticing that underemployment of people may well go along with some people being drastically overworked (Schor 1992).

An instructive situation is that of Europe after the financial crisis. It was trapped for years in a Pareto-suboptimal equilibrium with exceptionally low growth, low investment, high unemployment, and high vulnerability to financial shocks. As the president of the European Central Bank put it in those days: “we are in a situation now where you have large parts of the Euro area in what we call a bad equilibrium, namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios.” (Draghi 2012). In the mean time, another bad, if partially better equilibrium has emerged: Europe has entered a growth trajectory at a lower level and with much higher regional inequality than before the financial crisis of 2007.

In his analysis, Draghi has saddled a mechanism of self-fulfilling expectations with the responsibility for the economy selecting a “bad” equilibrium. Such expectations are not considered in the standard general-equilibrium modelling approach (and they need not be because current general equilibrium models do not allow for choosing from alternative futures). Of course, starting with Keynes (1936), Pigou (1927), and Kalecki (1933), their importance has often been pointed out, but so far neo-keynesian models have not led to an understanding of choices between alternative growth paths.

Game theory deals with decisions that agents take while looking for their best response to the expected behaviour of others. In terms of equilibrium selection, an interesting game to consider is the stag hunt (Rousseau 1974; Skyrms 2001). A stag hunt is a game with two pure Nash equilibria, often one of them being risk dominant and the other one payoff dominant. Cooperation of the players leads to the Pareto-superior equilibrium whereas a lack of trust results in a Pareto-inferior outcome (Harsanyi and Selten 1988). We will argue that the challenge of green growth cannot be understood in terms of the prisoner’s dilemma, which is often employed when mitigation activities are related to the tragedy of the commons, but rather in terms of the stag hunt.

Formulating the green-growth narrative in game-theoretic terms, Mielke and Steudle (2017) suggest to understand investors’ behaviour as a stag hunt. Coordination on the current brown growth path yields the risk dominant equilibrium, whereas coordination on a green growth path would be the payoff dominant, though riskier, option. This invites further research about triggers that can induce a transition from the risk-dominant to the cooperative state in the current situation, like e.g. in Mielke (2018 forthcoming).

In this paper, we focus on the task of establishing a simple proof-of-principle model which combines a Ramsey-type growth model with three critical building blocks – to be discussed below – in a game-theoretic format. Our model has multiple macro-economic equilibria, including a Pareto-inferior path of brown growth and a superior one of green growth. The choices of strategy in this game are investment and labour demand decisions. The payoffs to each player are utilities resulting from the interdependent decisions of all players. We will show that a stag-hunt-like structure can result for the payoff table. Then, the degree of trust and cooperation among the players determines whether the Pareto-optimal Nash equilibrium is obtained.

In previous work (Jaeger et al 2015; Tabara et al 2013), we found that adding learning by doing, a mechanism to account for self-fulfilling expectations and a non-standard labour market were sufficient in order to introduce meaningful win-win options in a standard computable general-equilibrium (CGE) model. We did so by inserting suitable parameters in the mathematical structure of a CGE model. Here we isolate these three mechanisms in a macro-economic growth model, so as to make their joint operation transparent.

The remainder of this paper is organised as follows: In Section 2, we give a short overview of the basic model structure. In Section 3, the building blocks of the green growth model are presented in more detail. In Section 4, we demonstrate that a model combining these building blocks can indeed be used to construct “investment games” with brown and green growth yielding a stag-hunt structure. Section 5 adds some conclusions.

2 The Basic Structure

Since this paper aims at understanding investment decisions in view of green growth, we need to consider some kind of inter-temporal utility maximisation. The standard approach is to define inter-temporal utility as the sum (or integral) of discounted momentary utilities, aka felicities, yielding a Ramsey-type growth model. We build on this approach and simplify things to the maximum for the sake of analytical clarity and easy tractability, but in such a way that generalisations in many directions will be possible.

In this spirit, we restrict ourselves to two time steps, present ($t = 0$) and future ($t = 1$). This will suffice to show the key mechanisms and their interaction. We also assume that felicity at time t depends on consumption C_t only (and not on leisure), and that the inter-temporal utility U is given by

$$U(C_0, C_1) = \ln C_0 + \rho \ln C_1. \tag{1}$$

where $0 < \rho < 1$ is the factor to discount future felicity.

Output is produced from capital K and labour L, the production technology is given by a production function $f(K, \eta L)$ with η for labour productivity. The problems with aggregate production functions are well-documented and usually ignored (for a recent discussion see Felipe and McCombie 2013). We use this analytical tool here for simplicity of exposition and to stay linked with the vast literature inspired by Ramsey. Disaggregating the production function will strengthen, not weaken our argument, as the role of multiple equilibria then increases. In fact, our argument is based on a first modest step of disaggregation by introducing two kinds of capital, brown and green (see Section 3.2).

Output can be used for either consumption or investment. The general structure then is

$$C_0 = f(K_0, \eta_0 L_0) - I_0 \tag{2}$$
$$K_1 = (1 - \delta) \cdot K_0 + I_0 \tag{3}$$
$$C_1 = f(K_1, \eta_1 L_1) \tag{4}$$

There are a given initial capital stock K_* and a given labour force L_* such that the actual K_0 as well as L_0 and L_1 satisfy:

$$K_0 \leq K^*, L_0 \leq L^*, L_1 \leq L^* \tag{5}$$

In the basic case, utility maximisation leads to full utilisation of the capital stock and the labour force. However, unemployment is a major feature of actual economies and will play a key role in the analysis of brown vs. green growth. Underutilisation of capital stocks may be relevant, too, but as it would not change the main argument, we will not consider it here.

All variables are bounded to be non-negative. In particular, we do not allow for negative investment, that is, investment is irreversible. The production function is assumed to be well-behaved, in particular homogeneous of degree one.[2] At time $t = 0$, output is used for consumption C_0 and investment I_0 (Equation (2)). The capital stock decays at rate δ and is increased by investments (Equation (3)). Since we consider only two time steps for calculating overall utility, investments at time $t = 1$ are zero: their benefits would not matter for utility while they would be at the expense of consumption at $t = 1$, which does matter (Equation (4)). Decision variables (control variables) are I_0, L_0, L_1. The equations will be refined later on, in particular there will be two types of capital (green capital G_t and brown capital B_t), and η_t will change endogenously through learning by doing, but the model structure remains firmly based on the Ramsey-style picture of (1)-(4).

[2] Dropping this standard assumption will require generalising the present analysis to models of imperfect competition, from monopolistic competition to the more realistic, but thorny instances of oligopoly. We leave this for future research.

2.1 Households and Firms

Equations (2)-(4) refer to aggregates for the economy as a whole. Now let's assume an economy of m households owning n firms. As customary in basic economic models, we assume these can be modelled as populations of representative (i.e. identical) households and firms.[3] Each household has shares for $\frac{1}{m}$ of each firm. Firms in turn own $\frac{1}{n}$ of the capital stock of the economy (Barro and Sala-i-Martin 2004, p.152). Initially, there are as many shares as units of initial capital, so we can write the initial wealth of household i as $\frac{K_0}{m} = K_0^{(i)}$. Households consume part or all of their income and invest the rest by getting a number of additional shares equal to the quantity of capital invested. Therefore, the wealth of household i at time 1 is $\frac{K_1}{m} = K_1^{(i)}$.

Under these conditions, the optimisation problem has to be written differently. Household i now earns a wage income w per unit of labour it performs, a property income from the return of r per share it owns, and a fraction $\frac{1}{m}$ of the profit (or deficit), Π_t, made in the economy.[4] For household i we get

$$C_0^{(i)} = r_0 K_0^{(i)} + w_0 L_0^{(i)} + \Pi_0^{(i)} - I_0^{(i)} \tag{6}$$

$$K_1^{(i)} = (1-\delta) \cdot K_0^{(i)} + I_0^{(i)} \tag{7}$$

$$C_1^{(i)} = r_1 K_1^{(i)} + w_1 L_1^{(i)} + \Pi_1^{(i)} \tag{8}$$

$$\Pi_t^{(i)} = \Pi_t \frac{K_t^{(i)}}{K_t} \tag{9}$$

with the goal of maximising overall utility

$$U^{(i)}(C_0^{(i)}, C_1^{(i)}) = \ln C_0^{(i)} + \rho \ln C_1^{(i)} \tag{10}$$

For firm j, the relevant goal is to maximise

$$\Pi_t^{(j)} = f(K_t^{(j)}, \eta_t L_t^{(j)}) - r_t K_t^{(j)} - w_t L_t^{(j)}, \quad t = 0, 1, \tag{11}$$

with $\Pi_t^{(j)}$ being the firm's profit at times t.

Furthermore, market clearing implies

$$\sum_{i=1}^{m} K_t^{(i)} = \sum_{j=1}^{n} K_t^{(j)} \quad \text{(capital market)} \tag{12}$$

$$\sum_{i=1}^{m} L_t^{(i)} = \sum_{j=1}^{n} L_t^{(j)} \quad \text{(labour market)} \tag{13}$$

$$\sum_{i=1}^{m} \left(C_t^{(i)} + I_t^{(i)} \right) = \sum_{j=1}^{n} f(K_t^{(j)}, \eta_t L_t^{(j)}) \quad \text{(goods market).} \tag{14}$$

and we have

$$\Pi_t = \sum_{j=1}^{n} \Pi_t^{(j)} \tag{15}$$

If (11) reaches a maximum with positive production, we must have

$$r_t = \frac{\partial f}{\partial K_t^{(j)}} \quad \text{and} \quad w_t = \frac{\partial f}{\partial L_t^{(j)}} \tag{16}$$

[3] Kirman (1992) is still the canonical analysis of the problems involved in the construction of representative economic agents. While many economic claims break down without that construction, there is no reason to expect this for our results concerning green growth. Further work shall explore the hypothesis that our results will even be strengthened, rather than weakened, by introducing heterogeneous agents.

[4] As usual in the economics literature (less so in accounting), we do not call $r_t K_t$ profit but return on capital, otherwise we would have to call Π_t extra-profit.

(14) can be rewritten as

$$\sum_{j=1}^{n} f(K_t^{(j)}, \eta_t L_t^{(j)}) - \Pi_t = \sum_{i=1}^{m} \left(r_t K_t^{(i)} + w_t L_t^{(i)} \right) = \sum_{j=1}^{n} \left(r_t K_t^{(j)} + w_t L_t^{(j)} \right) \tag{17}$$

which, as all firms are equal, results in

$$f(K_t^{(j)}, \eta_t L_t^{(j)}) = r_t K_t^{(j)} + w_t L_t^{(j)} + \Pi_t^{(j)} \tag{18}$$

Combining (16) and (18) results in

$$f(K_t^{(j)}, \eta_t L_t^{(j)}) = \frac{\partial f}{\partial K_t^{(j)}} \cdot K_t^{(j)} + \frac{\partial f}{\partial (\eta_t L_t^{(j)})} \cdot (\eta_t L_t^{(j)}) + \Pi_t^{(j)} \tag{19}$$

Given a well-behaved production function, we have $\Pi_t^{(j)} = 0$.

2.2 Optimisation and the Investment Game

If all households and all firms are equal, a benevolent planner will simply optimise the utilities $U^{(i)}$ for all households, i.e. she will choose $I_0 = \sum_{i=1}^{m} I_0^{(i)}, L_0 = \sum_{j=1}^{n} L_0^{(j)}, L_1 = \sum_{j=1}^{n} L_1^{(j)}$ of equations (2)-(4) such that the utility given in (1) is maximised, which is equivalent to a maximisation of the sum of all utilities given in (10).

If there is no central planner, the question of how prices (r_t and w_t) come about has to be addressed. Solving this problem in a way that market clearing is obtained without some price-setting "auctioneer" is far beyond the scope of this paper. In the model discussed here, the players are price takers, and prices are given so that markets clear.

The situation can be looked at as a game with three kinds of players: the auctioneer, representative households and representative firms. Each player understands the whole game, including goals and constraints of the other players. The auctioneer is a "price player" (Levin 2006), her strategies are tuples (w_0, r_0, w_1, r_1) and her payoffs are maximal if and only if the supplies and demands of households and firms match, i.e. iff markets clear. Households and firms maximise (10) and (11) given the prices set by the auctioneer. If the game has a single Nash equilibrium, that's what their choices will result in.

The total set of decision variables of the household are its supply of labour and capital, $I_0^{(i)}, I_1^{(i)}, K_0^{(i)}, K_1^{(i)}$, with $I_0^{(i)} = K_1^{(i)} - K_0^{(i)}(1-\delta)$. Given the form of the utility function, at any positive wage the household will always supply its total labour force. Moreover, at any positive rate of return the household will also supply all the capital it owns, so the key decision variable of the household is investment $I_0^{(i)}$. Under these conditions, the problem of the household boils down to choosing the single amount of investment that will balance its marginal utility from present consumption with the marginal utility to be obtained in the future by additional investment. That requires $\frac{\mathrm{d}U^{(i)}}{\mathrm{d}I_0^{(i)}} = 0$.

As profits are zero, consumption can be written as

$$C_0^{(i)} = r_0 K_0^{(i)} + w_0 L_0^{(i)} - I_0^{(i)} \tag{20}$$

$$C_1^{(i)} = r_1((1-\delta)K_0^{(i)} + I_0^{(i)}) + w_1 L_1^{(i)} \tag{21}$$

And as the utility of household i is

$$U^{(i)}(C_0^{(i)}, C_1^{(i)}) = \ln C_0^{(i)} + \rho \ln C_1^{(i)}, \tag{22}$$

it follows that

$$\frac{\mathrm{d}U^{(i)}}{\mathrm{d}I_0^{(i)}} = 0 = -\frac{1}{r_0 K_0^{(i)} + w_0 L_0^{(i)} - I_0^{(i)}} + \frac{\rho r_1}{r_1 \left((1-\delta)K_0^{(i)} + I_0^{(i)} \right) + w_1 L_1^{(i)}}. \tag{23}$$

Thus for optimal investment we obtain

$$I_0^{(i)} = \frac{1}{(1+\rho)r_1}\left(\rho r_1 w_0 L_0^{(i)} - w_1 L_1^{(i)} + \rho r_1 r_0 K_0^{(i)} - r_1(1-\delta)K_0^{(i)}\right). \quad (24)$$

If, given the prices set by the auctioneer, the solution is negative, investment will be zero, because we have ruled out negative investment. Otherwise, there will be exactly one optimal positive amount of investment.

Each firm, in turn, chooses its decision variables, i.e. the demand for labour and capital, $L_0^{(j)}, L_1^{(j)}, K_0^{(j)}, K_1^{(i)}$, so as to maximise its profit $\Pi_t^{(j)}$ given the prices set by the auctioneer. As the firms are identical and together own the capital stock K, prices that would induce them to buy or sell capital cannot clear the capital market. The auctioneer will set prices so that firms are satisfied to operate with the capital they have while the sum of their labour demand will be equal to L_t. With a well-behaved production function there will be a single pair of prices yielding this result, and so this version of the investment game has a single Nash equilibrium, which is equivalent to a Walrasian market clearing equilibrium.

3 Three Building Blocks

In order to use the minimal Ramsey-type structure described in Section 2 to analyse the possibility of green growth, three building blocks are essential. Each one of them is well-established in the literature, but so far they have not been implemented together in one model. As we will see, this combination brings about remarkable results. In this section, they are first discussed one by one. Again, for the sake of clarity, they are introduced in the simplest possible way, but so as to allow for far-reaching generalisations.

3.1 Endogenous Technical Progress

First, we introduce endogenous technical progress with spillover effects through learning by doing[5]. The idea is that learning by doing goes hand in hand with capital accumulation, such that labour productivity η grows proportionally to the capital stock K. Then we have

$$\eta(K_t) = \frac{K_t}{K_0}\cdot\eta_0. \quad (25)$$

Furthermore, we assume a Cobb-Douglas production function

$$f(K, \eta L) = K^\alpha(\eta L)^{1-\alpha} \quad (26)$$

with $0 < \alpha < 1$. In combination with (25) this means

$$f(K_t, \eta_t L_t) = K_t^\alpha\left(\frac{K_t}{K_0}\eta_0 L_t\right)^{1-\alpha} = A\cdot K_t\cdot L_t^{1-\alpha} \quad (27)$$

with $A = \left(\frac{\eta_0}{K_0}\right)^{1-\alpha}$ being a constant factor depending on the initial parameters. This is the well-known AK model of technical progress. More sophisticated versions of endogenous technical progress are of course possible, but we do not need them at this stage.

[5] For endogenous growth models with spillover effects see e.g. Frankel (1962); Romer (1986); Lucas (1988), for empirical evidence about learning by doing see e.g. Nagy et al (2010).

3.1.1 Learning by Doing with a Benevolent Planner

As mentioned, with the utility given in (1), global utility maximisation always results in maximum employment. For simplicity we can therefore set $L_0 = L_1 = 1$ beforehand (assuming that the labour force does not grow or shrink). The basic equations to describe the aggregate system with endogenous technical change then are:

$$C_0 = K_0^\alpha \eta_0^{1-\alpha} - I_0 = A \cdot K_0 - I_0 \tag{28}$$

$$K_1 = (1-\delta)K_0 + I_0 \tag{29}$$

$$\eta_1 = \frac{K_1}{K_0} \cdot \eta_0 \tag{30}$$

$$C_1 = K_1^\alpha \eta_1^{1-\alpha} = K_1^\alpha \left(\frac{K_1}{K_0} \cdot \eta_0\right)^{1-\alpha} = A \cdot K_1. \tag{31}$$

From the necessary condition for utility maximisation, $\frac{\mathrm{d}U}{\mathrm{d}I_0} = 0$, optimal investment I_0 at time $t = 0$ follows as

$$I_0 = \frac{1}{1+\rho}\left(\rho A - 1 + \delta\right) K_0, \tag{32}$$

which is positive as long as $\rho A + \delta > 1$ (and $I_0 = 0$ otherwise, because we do not allow for negative investments).

3.1.2 Learning by Doing with Households, Firms, and Farms

As in Section 2.1, we next consider a situation with m identical households and n identical firms. Let $K_t = \sum_{i=1}^m K_t^{(i)} = \sum_{j=1}^n K_t^{(j)}$ denote the aggregate capital stock at time t. Household i again maximises its utility $U^{(i)} = \ln C_0^{(i)} + \rho \ln C_1^{(i)}$ subject to

$$C_0^{(i)} = r_0 K_0^{(i)} + w_0 L_0^{(i)} + \Pi_0^{(i)} - I_0^{(i)} \tag{33}$$

$$K_1^{(i)} = (1-\delta) \cdot K_0^{(i)} + I_0^{(i)} \tag{34}$$

$$C_1^{(i)} = r_1 K_1^{(i)} + w_1 L_1^{(i)} + \Pi_1^{(i)} \tag{35}$$

while firm j maximises its profit $\Pi_t^{(j)}$ at each time step

$$\max_{K_t^{(j)}, L^{(j)}} f\left(K_t^{(j)}, \frac{K_t}{K_0}\eta_0 L_t^{(j)}\right) - r_t K_t^{(j)} - w_t L_t^{(j)}. \tag{36}$$

As commonly assumed in the literature, we take the effect on overall productivity by the investment of a single firm to be negligible. Therefore, firms consider $\frac{\partial K_t}{\partial K_t^{(j)}} \approx 0$, and thus they choose capital and labour according to

$$r_t = \frac{\partial f}{\partial K^{(j)}} = \alpha \cdot \frac{f\left(K_t^{(j)}, \frac{K_t}{K_0}\eta_0 L_t^{(j)}\right)}{K_t^{(j)}} \tag{37}$$

$$w_t = \frac{\partial f}{\partial L^{(j)}} = (1-\alpha) \cdot \frac{f\left(K_t^{(j)}, \frac{K_t}{K_0}\eta_0 L_t^{(j)}\right)}{L_t^{(j)}} \tag{38}$$

This results in too low an investment. In our case it leads to

$$I_0^{(i)} = \frac{1}{1+\alpha\rho}\left(\alpha\rho A - 1 + \delta\right) K_0^{(i)}, \tag{39}$$

which is smaller than the optimal investment of $\frac{1}{m} I_0$ for the I_0 of the benevolent planner's case as given in (32). This means that the growth rate in the decentralised model is smaller than in the case of a central planner and the resulting equilibrium is sub-optimal. In the language of general equilibrium theory and its many relatives, this non-optimality is due to the fact that learning-by-doing with the assumed spillovers is an externality. In theory, the social optimum could be obtained by introducing a consumption tax that is used to subsidise investment or production (e.g. Barro and Sala-i-Martin 2004, p. 216f).

In game-theoretical terms, the resulting structure is a prisoner's dilemma that rewards free riding. To keep the analysis simple, we imagine a world of independent farmers where households and firms coincide. We then have n household-firms, i.e. farms, producing goods from their own capital and labour. The relevant equations for farm i are

$$C_0^{(i)} = \left(K_0^{(i)}\right)^{\alpha} \left(\eta_0 L_0^{(i)}\right)^{1-\alpha} - I_0^{(i)} \tag{40}$$

$$K_1^{(i)} = (1-\delta) \cdot K_0^{(i)} + I_0^{(i)} \tag{41}$$

$$\eta_1 = \frac{K_1}{K_0} \cdot \eta_0 \tag{42}$$

$$C_1^{(i)} = \left(K_1^{(i)}\right)^{\alpha} \left(\eta_1 L_1^{(i)}\right)^{1-\alpha} \tag{43}$$

with the utility $U^{(i)} = \ln C_0^{(i)} + \rho \ln C_1^{(i)}$ to be maximised.

If all farms are equal, i.e. $K_0^{(i)} = \frac{1}{n} K_0$ and $L_0^{(i)} = L_1^{(i)} = \frac{1}{n}$, the benevolent planner's result for the optimal investment of farm i is

$$I_{bp}^{(i)} = \frac{1}{1+\rho} \left(\rho A - 1 + \delta\right) K_0^{(i)} = \frac{1}{1+\rho} \left(\rho A - 1 + \delta\right) \frac{K_0}{n}. \tag{44}$$

However, our farms maximise only their own utility $U^{(i)}$. If a single farm tried to maximise overall welfare, it would actually reduce its individual utility – this is the point of a prisoner's dilemma. To solve the utility maximisation problem subject to (40)-(43), farm i has to make assumptions about the investment of all others because for the productivity η_1 the sum of investments of all farms matter (see (42)). It can be easily shown that if farm i assumes everybody to invest according to the benevolent planner's plan, it maximises its own utility investing

$$I_{fr}^{(i)} = \frac{1}{1+\alpha\rho} \left(\alpha\rho A - 1 + \delta\right) K_0^{(i)} < I_{bp}^{(i)}, \tag{45}$$

which is exactly the result of (39). This means, if farm i assumes everyone else to invest according to (44), the optimal strategy for farm i is to invest less and "free ride" on the technological progress triggered by the higher investment of the others. Furthermore, it can be shown that even in case the farm assumes everybody to invest according to (45), it will still be optimal for farm i to invest according to (45). However, now utility is now lower than in the previous case because there is no free-riding effect anymore.

To illustrate this, let us set up a game of farms with two possible investment strategies, S_{bp} which leads to investing I_{bp} (cooperate) and S_{fr} which leads to investing only $I_{fr} < I_{bp}$ (defect). The utilities for farm i in a game in which the other farms $j \neq i$ either cooperate or defect yield the following prisoner's dilemma[6]:

3.2 Green and Brown Capital

In order to model emissions dynamics, we now introduce two types of capital: "green" (low emissions), G, and "brown" (high emissions), B. Of course, one can introduce more than two types, including zero and even negative emissions. The key point is that the different

[6] Parameters used are $K_0^{(i)} = 1$, $L_{0,1}^{(i)} = 1$, $\eta_0 = 1$, $\delta = 0.05$, $\alpha = 0.2$, $\rho = 0.99$, and $\sum_{j \neq i} K_0^{(j)} \gg K_0^{(i)}$.

types of capital differ in their emissions intensities and that learning by doing is specific to types of capital.[7]

Production from green or brown capital and labour is given by Cobb-Douglas functions $f(G,\gamma g) = G^\alpha(\gamma g)^{1-\alpha}$ and $f(B,\beta b) = B^\alpha(\beta b)^{1-\alpha}$ with $0 < \alpha < 1$, respectively, where γ and β are green and brown labour productivities, and g and b are the quantities of labour working with green and brown capital. We assume that both technologies produce the same good, which can then be either consumed or invested into brown or green capital stock. Furthermore, we assume that both capital stocks decay with equal rate δ.

For given capital stocks G, B and productivities γ, β the allocation of a constant amount of labour $g + b$ is optimal – in the sense that production yields a maximum – if

$$\frac{g}{b} = \frac{G}{B}\left(\frac{\gamma}{\beta}\right)^{\frac{1-\alpha}{\alpha}}. \tag{46}$$

Which technology to invest in is optimal (in terms of utility optimisation) then depends on the initial capital stocks and the respective productivities.

In order to discuss key properties of the two-capital-types structure we again consider an example with n farms, i.e. n households/firms producing goods from their own capital and labour. In the beginning, all farms are equal in the sense that they own identical shares of the initial capital stocks $G_0^{(i)} = \frac{1}{n}G_0$ and $B_0^{(i)} = \frac{1}{n}B_0$, and mobilise $g_t^{(i)} + b_t^{(i)} = \frac{1}{n}$ quantities of labour at time steps $t = 0, 1$. Farm i then maximises its utility $U^{(i)} = \ln C_0^{(i)} + \rho \ln C_1^{(i)}$ subject to

$$C_0^{(i)} = f(G_0^{(i)}, \gamma_0 g_0^{(i)}) + f(B_0^{(i)}, \beta_0 b_0^{(i)}) - I_G^{(i)} - I_B^{(i)} \tag{47}$$

$$G_1^{(i)} = (1-\delta)\cdot G_0^{(i)} + I_G^{(i)} \tag{48}$$

$$B_1^{(i)} = (1-\delta)\cdot B_0^{(i)} + I_B^{(i)} \tag{49}$$

$$C_1^{(i)} = f(G_1^{(i)}, \gamma_1 g_1^{(i)}) + f(B_1^{(i)}, \beta_1 b_1^{(i)}). \tag{50}$$

Without knowing the initial capital stocks G_0, B_0 and the productivities γ_t, β_t not much can be said about optimal investment strategies. If we assume endogenous technical progress through learning by doing as developed in Section 3.1, however, the structure of a coordination game results, as illustrated in the following example.

Consider a simple game of many farms in which every farm invests half of its production, i.e. $I_{G/B}^{(i)} = \frac{1}{2}\left(f(G_0^{(i)}, \gamma_0 g_0^{(i)}) + f(B_0^{(i)}, \beta_0 b_0^{(i)})\right)$, and consumes the other half. Each farm chooses one of two strategies: it can invest either only into its brown or only into its green capital stock. Endogenous technical progress as in Section 3.1 now yields:

$$\gamma_1 = \frac{\sum_{i=1}^n G_1^{(i)}}{\sum_{i=1}^n G_0^{(i)}}\cdot\gamma_0 \quad \text{and} \quad \beta_1 = \frac{\sum_{i=1}^n B_1^{(i)}}{\sum_{i=1}^n B_0^{(i)}}\cdot\beta_0. \tag{51}$$

Let each farm start with two equal initial capital stocks and productivities. Figure 1 shows the payoffs for the resulting game[8], i.e. the utilities of a farm investing into green and a farm investing into brown capital versus the ratio of farms investing into green capital. It is always better to do what most of the others do: directed technical change leads to a coordination game.

Our first building block was an investment game involving the prisoner's dilemma of induced technical change. To address it in practice would require additional taxes, which is usually politically difficult, even more so with a global problem like climate change that relies on international coordination of those taxes. Moreover, the taxes have to be permanent,

[7] Our approach here relies on the work on directed technical change pioneered by Acemoğlu, both in general (Acemoğlu 2002) and in view of green growth (Acemoğlu et al 2016, 2012). Aghion et al (2016) apply this approach to technical change in the car industry, Pottier et al (2014) highlight some pitfalls.

[8] Parameters used are $G_0^{(i)} = B_0^{(i)} = 1$, $\gamma_0 = \beta_0 = 2$, $\delta = 0.01$, $\alpha = 0.2$, $\rho = 1$.

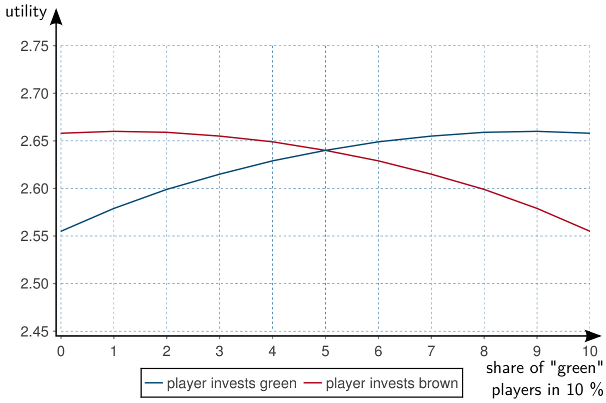

Fig. 1 Utility of one player investing into green (blue line) or brown (red line) depending on the overall share of farms investing into green. It pays off to do what most of the players do, and utilities are the higher the more farms play the same strategy.

because otherwise the economy will fall back into the Pareto inferior Nash equilibrium. Subsidies may be politically easier to introduce, but they still require international coordination and long-term financing.

Our second building block added a coordination-game structure to the investment game, meaning that the green equilibrium would be stable without permanent policy measures. But starting from the brown equilibrium like the one the world economy is presently in, it does not pay for an agent to move individually towards the green equilibrium. And even if the agents moved together, there would be a transition cost in order to build up γ until it catches up to the level of β (that centuries of brown growth have established).

Therefore, quite stringent policy measures would be necessary to trigger and achieve the transition from brown to green growth. Acemoğlu et al (2012) argue that a combination of taxes on brown goods and subsidies for green R&D would do the trick; most likely additional regulations would be needed as well. These would be transitory measures and they are probably easier to realise than the permanent measures required for the first building block in isolation, but politically they are still far from trivial.

The third building block will modify the investment game in ways that allow for win-win strategies at a large scale. This makes it essential for green growth.

3.3 Labour Market

The third and last building block needed to understand the transition from brown to green growth is the matching process on labour markets (Mortensen and Pissarides 2011). The success of a firm depends to a considerable extent on the fit between the tasks arising in the firm and the skills and capabilities of the different people involved in it. Ideally, a firm will dedicate resources to improving that fit so as to get an optimal balance between what one may call productive activities and search activities (where the latter will include training and supervision). For each firm this balance depends on the effectiveness of the search effort. However, this effectiveness depends on the search efforts of all the firms together: if firms search hard, they will hire more, reducing the pool in which they search and thereby lowering search effectiveness. This creates an externality with two key characteristics. Firstly, it creates a continuum of equilibria on the labour market, ranging from low to high search efforts (Howitt and McAfee 1987; Krause and Lubik 2010). Secondly, attempts to internalise the external effect would generate transaction costs quickly exceeding the benefits to be expected (Farmer 2013). For example, imagine a government trying to establish a scheme taxing search efforts when they are too high, subsidising them when they are too low and leaving them to their own devices in between. The costs of implementing such a scheme – including the costs for a government trying to identify an optimal search level as well as defining what counts as a search effort in the first place – would become prohibitive long before having realised even a second-best solution. An analogous argument holds for the search efforts of households, but as we have simplified things by keeping labour out of utility functions, we can neglect this here.

The matching problem in labour markets raises the question of equilibrium selection on these markets, and the answer depends on what drives the search efforts of firms. Not the only, but certainly an essential factor are animal spirits of investors, introduced by Keynes as an indispensable element of analysis. With a whole range of contributions, Roger Farmer has shown how to relate them to the matching process on labour markets.[9] The main finding is the existence of a continuum of equilibria, with equilibrium selection shaped to a large extent through the collective mood, the animal spirits (Farmer formalises them through a "belief function") of investors.

Moving to modelling against this background, let total labour employed, L, be the sum of productive labour x and search labour s, where search labour determines the amount of total labour, and total labour is bounded:

$$L = x + s \tag{52}$$

$$L = \phi(s) = (\mathit{\Gamma} s)^{\frac{1}{2}} \tag{53}$$

$$L \leq 1 \tag{54}$$

Without search, there is no work, with all labour dedicated to search, no production. $0 < \mathit{\Gamma} \leq 2$ is a parameter for the overall effectiveness of search labour (i.e. independent of market conditions). It is bounded above because labour is bounded above as well. From (52) and (53) we get

$$x = L\left(1 - \frac{L}{\mathit{\Gamma}}\right). \tag{55}$$

With representative agents, the ratio of productive labour to total labour is the same for all firms and equals the aggregate ratio:

$$\frac{x^{(j)}}{L^{(j)}} = \frac{x}{L} = 1 - \frac{L}{\mathit{\Gamma}}. \tag{56}$$

For a standard Cobb-Douglas production function $f(K, \eta x) = K^{\alpha}(\eta x)^{1-\alpha}$, $0 < \alpha < 1$, aggregate production in terms of K and total labour L can be written as

$$f(K, \eta L) = K^{\alpha}\left(\eta L\left(1 - \frac{L}{\mathit{\Gamma}}\right)\right)^{1-\alpha}, \tag{57}$$

and for firm j results

$$f(K^{(j)}, \eta L^{(j)}) = (K^{(j)})^{\alpha}\left(\eta L^{(j)}\left(1 - \frac{L}{\mathit{\Gamma}}\right)\right)^{1-\alpha}. \tag{58}$$

Since $L = \sum_{j=1}^{n} L^{(j)}$ there is a labour search externality: if all firms increase their search effort, their individual effort becomes less effective, but the contribution of an individual firm to search effectiveness is negligible. To analyse the implications of this externality, we now compare the benevolent planner with individual agents.

[9] E.g. Farmer et al (2009); Farmer (2011, 2013).

3.3.1 Benevolent Planner

The benevolent planner would choose L_0, L_1 and I_0 so as to maximise the overall utility $U = \ln C_0 + \rho \ln C_1$ subject to

$$C_0 = K_0^\alpha \left(\eta_0 L_0 \left(1 - \frac{L_0}{\Gamma}\right)\right)^{1-\alpha} - I_0 \tag{59}$$

$$K_1 = (1-\delta) \cdot K_0 + I_0 \tag{60}$$

$$C_1 = K_1^\alpha \left(\eta_1 L_1 \left(1 - \frac{L_1}{\Gamma}\right)\right)^{1-\alpha} \tag{61}$$

Using the constraints to substitute C_0 and C_1 in the utility function and setting the partial derivatives with regard to labour and investment to zero we get:

$$L_0 = L_1 = \frac{\Gamma}{2} \tag{62}$$

$$I_0 = \frac{\alpha\rho}{1+\alpha\rho} Q_0 - \frac{1-\delta}{1+\alpha\rho} K_0. \tag{63}$$

Substituting $Q_0 = f(K_0, \eta_0 L_0)$ and $L_0 = \frac{\Gamma}{2}$ yields

$$I_0 = \frac{\alpha\rho}{1+\alpha\rho} K_0^\alpha \left(\eta_0 \frac{\Gamma}{4}\right)^{1-\alpha} - \frac{1-\delta}{1+\alpha\rho} K_0 \tag{64}$$

This characterises the single Pareto optimum of the system. Notice that as long as $\Gamma < 2$, labour demand in the Pareto optimum is smaller than labour supply: there still is unemployment. The reason is that households keep offering the whole available labour (they expect to increase their income at given wages), but employing all of it would imply so much search labour by the firms that productive labour would be smaller than in the Pareto optimum.

3.3.2 Firms and Households

In the case with n equal firms and m equal households, firms maximise their individual profits. The profit of firm j is given by

$$\Pi_t^{(j)} = Q_t^{(j)} - w_t^{(j)} L_t^{(j)} - r_t K_t^{(j)} \quad \text{with} \tag{65}$$

$$Q_t^{(j)} = \left(K_t^{(j)}\right)^\alpha \left(\eta_t L_t^{(j)} \left(1 - \frac{L_t}{\Gamma}\right)\right)^{1-\alpha}. \tag{66}$$

Notice that in the production function for the individual firm two different labour variables appear: L_t^j and L_t, while in the production function for the benevolent planner there was only L_t.

Because firms are equal and $L_t = \sum_{j=1}^n L_t^{(j)}$, we have $\frac{Q_t^{(j)}}{L_t^{(j)}} = \frac{Q_t}{L_t}$ and $\frac{Q_t^{(j)}}{K_t^{(j)}} = \frac{Q_t}{K_t}$ and so, setting the partial derivatives of profit with regard to the firm's labour and capital to zero, we get:

$$r_t = \alpha \frac{Q_t}{K_t} \quad \text{and} \quad w_t = (1-\alpha) \frac{Q_t}{L_t}. \tag{67}$$

Household i maximises its utility $U^{(i)} = \ln(C_0^{(i)}) + \rho \ln(C_1^{(i)})$ subject to

$$C_0^{(i)} = r_0 K_0^{(i)} + w_0 L_0^{(i)} - I_0^{(i)} \tag{68}$$

$$K_1^{(i)} = (1-\delta) K_0^{(i)} + I_0^{(i)} \tag{69}$$

$$C_1^{(i)} = r_1 K_1^{(i)} + w_1 L_1^{(i)} \tag{70}$$

Again, using the constraints to substitute C_0 and C_1 in the utility function and setting the partial derivatives with regard to labour and investment to zero, we now get:

$$I_0^{(i)} = \frac{1}{1+\rho}\left(\rho(r_0 K_0^{(i)} + w_0 L_0^{(i)}) - (1-\delta)K_0^{(i)} - \frac{w_1 L_1^{(i)}}{r_1}\right) \tag{71}$$

With w_1 and r_1 according to (67) and $L_0^{(i)} = L_1^{(i)} = \frac{\Gamma}{2m}$ this is equivalent to (64). However, for reasons explained in the discussion of the benevolent planner, the representative household will supply $\frac{L_t}{m}$ rather than $\frac{\Gamma}{2m}$. The Pareto optimum has measure zero in the set of possible equilibria, but even getting close to it seems difficult: as discussed in the introduction, employment in the world economy is far below any plausible candidate for Pareto optimality.[10]

This also means that the auctioneer cannot pursue market clearing in all markets, but only in the product and capital markets. With the resulting degree of freedom, the auctioneer may set prices so that investment by households and demand for labour and capital by firms correspond to the Pareto optimum. But there is a whole continuum of prices for which product and capital markets clear while unemployment prevails in the labour market. We get a coordination game with a continuum of labour market equilibria. The ones with high employment are related to low search efficiency, i.e. $(1 - \frac{L_t}{\Gamma})$ is small, whereas in a low employment equilibrium, search is more effective.

Under these conditions, supply and demand are insufficient to determine an equilibrium. Supply and demand can only operate once some other mechanism has selected an equilibrium in whose neighbourhood they can be meaningfully specified. A key candidate for such a mechanism are the proverbial "animal spirits", understood as a pattern of collective expectations sustained by social norms. Combining the three building blocks – learning by doing, green and brown capital, and the matching process on the labour market – leads to the possibility that green growth could be primarily a matter of fostering norms and expectations likely to select a green growth equilibrium rather than the present brown one. The statement on climate policy by corporations quoted in the Introduction (Apple et al 2017) is perhaps best read in such terms.

This is not to say that taxes, subsidies and other policy measures are not relevant to green growth, quite the opposite: they may be relevant both as direct financial incentives, and also as policy signals needed to establish norms and expectations that will drive equilibrium selection from brown to green growth. The underlying structure is no more a prisoner's dilemma but a coordination game.

In the next section, we illustrate the coordination-game structure with a numerical example.

4 A Stag Hunt for Green Growth

By combining the three components described in the previous section, it is now possible to understand how the transition from a brown growth to a green growth equilibrium can result in a Pareto improvement.

In Section 1, we suggested the need for macro-economic models that can describe multiple-equilibria situations in which not all equilibria are Pareto optimal. It is worth reiterating that the historically given equilibrium of the world economy is far from Pareto optimal, for at least two reasons. First, the spillovers from learning-by-doing lead to suboptimal investment, missing opportunities for capital accumulation coupled with technical progress (as we described in an abstract way in Section 3.1). This holds globally, but also in many

[10] As a technicality, we remark that labour markets without any unemployment could arise only if $\Gamma = 2$ and by chance the firms would dedicate half the labour force to search labour.

Scenario	Model implementation	Real world interpretation
brown	- low investment - low labour demand - mostly brown capital stock	- investment weakness - unemployment - unsustainably high emissions
green	- higher investment - higher labour demand - shift towards green capital	- investment need (in new technologies) - higher labour demand due to higher growth - capital shift to low-emission technologies

Table 1 Scenarios in the proof-of-principle model with interpretations.

national and regional economies.[11] Second, the labour market indeterminacy generated by the matching problem is resolved in reality by expectations rooted in social norms. These expectations usually leave considerable parts of the labour force underutilised. Again, this holds at the global scale as well as for most national and regional economies. Therefore, in the following we assume an equilibrium characterised by less than optimal investment rates and levels of employment. Although we do not explicitly consider emissions in the model, we call this equilibrium the brown equilibrium due to the fact that the current state of the world economy is characterised by an unsustainable use of resources.

A shift towards green growth can address both shortcomings. The increased investment required for the buildup of green capital induces additional learning by doing, allowing to achieve a higher growth rate. Moreover, additional people can be hired for green labour, increasing employment. Employment augments even more because additional search labour is needed to address the resulting matching problem. Against this background, the higher growth achieved in the transition from brown to green growth may then be stabilised. For this purpose, however, the norm based expectations of the brown past must shift towards the green future. Economic incentives like a robust carbon price and subsidies for green R&D may play a key role in this shift. More importantly, the actual experience of a higher growth rate in a green growth equilibrium is likely to be crucial in order to stabilise new expectations.

Having these green and brown growth scenarios in mind, we illustrate the combined effect of the three building blocks introduced in Section 3. Table 1 gives an overview of the links between the model building blocks and the brown and green growth real-world narratives, respectively.

In the next part of this section, we construct an investment game using a model that combines these three components. For this we need to define the players of the game, the sets of their possible strategies, and their payoffs. For the latter it seems straightforward to use players' utilities but other choices (e.g. profits) could be reasonable as well.

In Section 1, we introduced the stag hunt as a game that is of interest in the context of possible win-win options due to its two Nash equilibria of which only one is Pareto-optimal. In order to obtain it, some coordination of the players is essential and the Pareto-inferior equilibrium can be the risk-dominant one.

In the following, we demonstrate how our building blocks can be used to construct an investment game which exhibits such a stag-hunt-like structure. Of course, the building blocks of Section 3 by themselves do not force such a structure, many other games with a lot of different features can be built based on them. Our point is to show that a stag-hunt

[11] In emerging economies like China, the problem is more subtle, as very high levels of investment are concentrated on a few sectors – especially real estate – where they outpace current demand. Analysing the policy implications of this situation goes beyond the scope of the present paper, but it is worth noticing that a shift to a green growth path offers an opportunity for better diversification of investment.

strategy of player i	brown investment by player i	green investment by player i	search labour related with strategy choice
S_b (brown)	$I_B^{(i)} = I_B > 0$	$I_G^{(i)} = 0$	$s_1^{(i)} = \frac{1}{2}s_0$
S_g (green)	$I_B^{(i)} = 0$	$I_G^{(i)} = I_G > I_B$	$s_1^{(i)} > \frac{1}{2}s_0$.

Table 2 Investment strategies available to both players.

like structure – and thus a structure that allows for win-win options – can be obtained from a growth model with these three components.

4.1 A Green Investment Game

In this example, we look at a two-player game. As in the previous sections, we consider only two time steps, $t = 0$ and $t = 1$, so there is an investment decision to be taken merely at $t = 0$. The players here are households that own a green and a brown capital stock which they rent out to a firm. The firm produces a good, which can be used for consumption or investment (in both the green or the brown capital stock), from these capital stocks and from labour which is also supplied by the two households. The players' payoffs are their utilities; as before, for player i the utility is is given by $U^{(i)} = \ln C_0^{(i)} + \rho \ln C_1^{(i)}$.

There are two strategies available to the players, the "green" strategy S_g and the "brown" strategy S_b. To choose S_g means that the household chooses to invest only in its green capital stock, to choose S_b means that it invests only in brown. We assume that total labour L_0 employed by the firm initially is given (i.e. initial search labour s_0 is given), and that both households always work an equal amount of time, i.e. $L_t^{(1)} = L_t^{(2)} = \frac{1}{2}L_t$. Additionally, we assume that in the beginning both players share the initial amount of brown and green capital equally, i.e. $G_0^{(1)} = G_0^{(2)} = G_0/2$ and $B_0^{(1)} = B_0^{(2)} = B_0/2$.

In the spirit of Table 1 we make the following further assumptions: The current state of the economy is represented by the brown scenario, and thus we assume that the amount of search labour s_0 is less than optimal in the beginning. Investments associated with the "green" strategy are higher than investments for "brown". Labour L_1 at $t = 1$ is supposed to follow from the players investment decisions in the following way: The firm's search labour s_1 is determined by two components depending on the players' strategy choice, $s_1 = s_1^{(1)} + s_1^{(2)}$. If household i chooses "brown" $s_1^{(i)} = \frac{1}{2}s_0$ (i.e. with brown investment, labour demand stays at the business-as-usual level), whereas if it chooses "green" the respective labour demand is higher, i.e. $s_1^{(i)} > \frac{1}{2}s_0$. This means that if everybody invests in brown, employment stays the same at $t = 1$ as for $t = 0$, while for investments in green, it increases.

Table 2 summarises the two possible strategies. Productive labour x_t then results from search labour s_t following (52), (53) as

$$x_t = \sqrt{\Gamma s_t} - s_t \tag{72}$$

and is assumed to be distributed optimally between technologies as given in (46).

So far we have only assumed that I_B is suboptimal, in line with the actual conditions of the world economy, and that $I_G > I_B$ because of the additional investment needed to build up a green capital stock. To keep things simple, we specify the two investment flows in analogy to equations (44) and (45):

$$I_G = \frac{1}{2} \cdot \frac{1}{1+\rho} \left[\rho Q_0 - (1-\delta)(G_0 + B_0)\right], \tag{73}$$

$$I_B = \frac{1}{2} \cdot \frac{1}{1+\alpha\rho} \left[\alpha\rho Q_0 - (1-\delta)(G_0 + B_0)\right] \tag{74}$$

where Q_0 is overall production from green and brown capital at $t = 0$. We then have $I_G > I_B$ because $0 < \alpha < 1$.

Capital stocks develop as

$$G_1^{(i)} = (1-\delta)\frac{G_0}{2} + I_G^{(i)} \tag{75}$$

$$B_1^{(i)} = (1-\delta)\frac{B_0}{2} + I_B^{(i)}, \tag{76}$$

and we assume endogenous technical progress through learning by doing as described above, which means that with $G_1 = G_1^{(1)} + G_1^{(2)}$ and $B_1 = B_1^{(1)} + B_1^{(2)}$ the productivities at $t = 1$ are

$$\gamma_1 = \frac{G_1}{G_0}\gamma_0 \tag{77}$$

$$\beta_1 = \frac{B_1}{B_0}\beta_0. \tag{78}$$

For the search labour we choose $s_0 < \frac{\Gamma}{4}$ (it follows from Equation (62) that $\frac{\Gamma}{4}$ would be the benevolent planner's search labour). If player i chooses S_g we set $s_1^{(i)} = \frac{\Gamma}{8}$, and we assume $s_1^{(i)} = \frac{1}{2}s_0 < \frac{\Gamma}{8}$ if S_b is chosen, meaning that, if both players choose "green", $s_1 = \frac{\Gamma}{4}$.

Last but not least, for the game we have to define how prices are set. We assume that wages are the same for all types of labour (search labour, green and brown productive labour), and that the wage is set as

$$w_t = (1-\alpha)\frac{Q_t}{L_t} \tag{79}$$

with Q_t being the overall (green and brown) production at time t. For the returns on green and brown capital we set

$$r_t^{(G)} = \alpha\frac{Q_t^{(G)}}{G_t} \tag{80}$$

$$r_t^{(B)} = \alpha\frac{Q_t^{(B)}}{B_t} \tag{81}$$

with $Q_t^{(G)}$ and $Q_t^{(B)}$ being the firm's green and brown production at time t. Income $Y_t^{(i)}$ of household i is then given by

$$Y_t^{(i)} = w_t L_t^{(i)} + r_t^{(G)} G_t^{(i)} + r_t^{(B)} B_t^{(i)} \tag{82}$$

and $Y_t^{(1)} + Y_t^{(2)} = Q_t$.

4.2 Results and Discussion

An implementation of the game described in Section 4.1 with the parameters $B_0 = 1.8$, $G_0 = 0.5$, $\beta_0 = 1.2$, $\gamma_0 = 0.6$, $s_0 = 0.3$, $\delta = 0.7$, $\alpha = 0.8$, $\rho = 0.99$, $\Gamma = 1.8$) yields the following payoff table:

	green	brown
green	-1.366 / -1.366	-1.396 / -1.383
brown	-1.383 / -1.396	-1.375 / -1.375

This game has two Nash equilibria, the "green" equilibrium (both players play green) and the "brown" equilibrium (both players play brown).

The green equilibrium is Pareto-superior to the brown equilibrium. However, if player 1 is not sure about what player 2 will do, he might not choose to play green, as the following consideration shows: if we assume that player 1 supposes player 2 to play green or brown with equal probability, his expected payoff for playing green is given by $E[\pi_G] = 0.5 \cdot (-1.366) + 0.5 \cdot (-1.396) = -1.381$ whereas $E[\pi_B] = -1.379 > E[\pi_G]$. In fact, the risk factor[12], for the green equilibrium is 0.55 (and thus 0.45 for the brown equilibrium). That means that player 1's expected payoff for green is larger than the expected payoff for brown only if he assumes player 2 to play green with a probability of more than 55%.

The aim of our considerations was to create a growth model with at least two equilibria that can represent a green and a brown growth path. The investment game presented in this section serves as a first example. Having multiple equilibria, the question that arises naturally is which of them will be selected. In our example, we have found a stag-hunt-like structure with two Nash equilibria, one of them payoff dominant and one risk dominant. In this context, the problem of equilibrium selection is far from easy. A discussion of different solution concepts can e.g. be found in Harsanyi and Selten (1988). For perfectly rational players it seems that they should automatically "agree" on the payoff-dominant equilibrium. However, this demands a certain amount of trust between the parties. Besides, any real-world setting involves a certain lack of rationality. Experimental work, e.g. by Van Huyck et al (1990), suggests that the secure option prevails in many situations. The question of risk dominance versus payoff dominance as an equilibrium selection concept in the context of green and brown investments has been discussed by Mielke and Steudle (2017).

5 Conclusion

We have combined three building blocks to a new kind of growth model. We used this model to define one-stage games, and studied the structure of these games.

Endogenous technical change through learning by doing with technological spillovers results in a game structure where there are incentives for one player to invest less than the amount a social planner would determine, and to "free-ride" on the improved productivity resulting from the higher investment by the others. If everybody follows this strategy, however, the payoffs for all players become lower than for the case in which everybody cooperates and invests according to what a benevolent planner would do. This is the game structure usually referred to as the prisoner's dilemma.

If there are two different types of capital and productivity spillovers happen only within each production technology, it is important that players coordinate on the technology they want to invest into. Generally speaking, payoffs are higher for all parties if they have a common strategy as to which of the two capital stocks to extend.

Furthermore, if a labour market with a continuum of equilibria – due to the matching problem – is added to this structure, we obtain a game which can result in a stag-hunt payoff structure. Unlike the prisoner's dilemma, it has two Nash equilibria, one of which is payoff dominant and the other risk dominant.

Many of the economic models used to study issues of sustainable development and green growth do not allow for multiple equilibria. By construction, therefore, they exclude the possibility of examining transitions between different economic equilibria. Our approach shows that a growth model combining the three building blocks of induced technical change, directed technical change and labour market indeterminacy can overcome this limitation.

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[12] The risk factor for the green equilibrium is the probability which player 1 has to assume for player 2 to play green in order to be indifferent what to play himself.

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